Trigonometry Worksheet - Year 12
(Answers)

Ex. B

(1)

(a) $1 - \cos^2 \frac{1}{2}\theta = \underline{\underline{\sin^2 \frac{1}{2}\theta}}$

(b) $5\sin^2 3\theta + 5\cos^2 3\theta = 5(\sin^2 3\theta + \cos^2 3\theta)$
$$= 5 \times 1$$
$$= \underline{\underline{5}}$$

(c) $\sin^2 A - 1 = -1(-\sin^2 A + 1)$
$$= -1(1 - \sin^2 A)$$
$$= -1 \times \cos^2 A$$
$$= \underline{\underline{-\cos^2 A}}$$

(d) $\dfrac{\sin\theta}{\tan\theta} = \sin\theta \div \tan\theta$
$$= \sin\theta \div \frac{\sin\theta}{\cos\theta}$$
$$= \cancel{\sin\theta} \times \frac{\cos\theta}{\cancel{\sin\theta}}$$
$$= \underline{\underline{\cos\theta}}$$

(e) $\dfrac{\sqrt{1-\cos^2 x}}{\cos x} = \dfrac{\sqrt{\sin^2 x}}{\cos x}$
$$= \frac{\sin x}{\cos x}$$
$$= \underline{\underline{\tan x}}$$

(f) $\dfrac{\sqrt{1-\cos^2 3A}}{\sqrt{1-\sin^2 3A}} = \dfrac{\sqrt{\sin^2 3A}}{\sqrt{\cos^2 3A}}$

$= \dfrac{\sin 3A}{\cos 3A}$

$= \underline{\underline{\tan 3A}}$

(g) $(1+\sin x)^2 + (1-\sin x)^2 + 2\cos^2 x$

$= 1 + \cancel{2\sin x} + \sin^2 x + 1 - \cancel{2\sin x} + \sin^2 x + 2\cos^2 x$

$= 2 + 2\sin^2 x + 2\cos^2 x$

$= 2 + 2(\sin^2 x + \cos^2 x)$

$= 2 + 2 \times 1$

$= \underline{\underline{4}}$

Note:- In the question above, I have used the short-cuts,

$(a+b)^2 = a^2 + 2ab + b^2$

and

$(a-b)^2 = a^2 - 2ab + b^2$

(h) $\sin^4\theta + \sin^2\theta\cos^2\theta = \sin^2\theta(\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta \times 1$

$= \underline{\underline{\sin^2\theta}}$

(i) $\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)^2$

$= 1^2$

$= \underline{\underline{1}}$

[Using the short-cut for $(a+b)^2$]

(2) $$2\sin\theta = 3\cos\theta$$

$(\div \cos\theta)$ $\qquad\qquad$ $(\div \cos\theta)$

$$\frac{2\sin\theta}{\cos\theta} = 3$$

$$2 \times \frac{\sin\theta}{\cos\theta} = 3$$

$$2\tan\theta = 3$$

$$\tan\theta = \frac{3}{2}$$

~~(3)~~ $\cancel{\sin x \cos y = 3\cos x \sin y}$

$\cancel{(\div \sin x \cos y)}$ $\qquad\qquad$ $\cancel{(\div \sin x \cos y)}$

$$\cancel{\frac{\sin x \cos y}{\sin x}}$$

(3) $$\sin x \cos y = 3\cos x \sin y$$

$(\div \cos x)$ $\qquad\qquad$ $(\div \cos x)$

$$\frac{\sin x}{\cos x} \times \cos y = 3\sin y$$

$$\tan x \cos y = 3\sin y$$

$(\div \cos y)$ $\qquad\qquad$ $(\div \cos y)$

$$\tan x = 3\,\frac{\sin y}{\cos y}$$

$$\tan x = 3\tan y$$

(4) (a) $\cos^2\theta = 1 - \sin^2\theta$

(b) $\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta}{1-\sin^2\theta}$

(c) $\cos\theta\tan\theta = \cancel{\cos\theta} \times \dfrac{\sin\theta}{\cancel{\cos\theta}}$

$= \sin\theta$

(d) $\dfrac{\cos\theta}{\tan\theta} = \cos\theta \div \tan\theta$

$= \cos\theta \div \dfrac{\sin\theta}{\cos\theta}$

$= \cos\theta \times \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta}$

$= \dfrac{1-\sin^2\theta}{\sin\theta}$

(e) $(\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = \cos^2\theta - \sin^2\theta$

$\cancel{= Co}$

$= 1 - \sin^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

(5)

(a) $$(\sin\theta + \cos\theta)^2 \equiv \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$$
$$\equiv (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$$
$$\equiv \underline{\underline{1 + 2\sin\theta\cos\theta}}$$

(b) $$\frac{1}{\cos\theta} - \cos\theta \equiv \frac{1}{\cos\theta} - \frac{\cos\theta}{1}$$
$$\equiv \frac{1 - \cos^2\theta}{\cos\theta}$$
$$\equiv \frac{\sin^2\theta}{\cos\theta}$$
$$\equiv \sin\theta \times \frac{\sin\theta}{\cos\theta}$$
$$\equiv \underline{\underline{\sin\theta \times \tan\theta}}$$

(c) $$\tan x + \frac{1}{\tan x} \equiv \frac{\sin x}{\cos x} + \frac{1}{\sin x/\cos x}$$
$$\equiv \frac{\sin x}{\cos x} + 1 \div \frac{\sin x}{\cos x}$$
$$\equiv \frac{\sin x}{\cos x} + 1 \times \frac{\cos x}{\sin x}$$
$$\equiv \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$\equiv \frac{\sin^2 x + \cos^2 x}{\sin x\cos x}$$
$$\cancel{\equiv \frac{1}{\sin x \cos x}} = \underline{\underline{\frac{1}{\sin x\cos x}}}$$

(d) $\cos^2 A - \sin^2 A \equiv \cos^2 A - (1 - \cos^2 A)$

$\equiv \cos^2 A - 1 + \cos^2 A$

$\equiv \underline{\underline{2\cos^2 A - 1}}$

$\cos^2 A - \sin^2 A \equiv (1 - \sin^2 A) - \sin^2 A$

$\equiv 1 - \sin^2 A - \sin^2 A$

$\equiv \underline{\underline{1 - 2\sin^2 A}}$

$\therefore \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$ //

(e) $(2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2$

$\equiv 4\sin^2\theta - \cancel{4\sin\theta\cos\theta} + \cos^2\theta + \sin^2\theta + \cancel{4\sin\theta\cos\theta} + 4\cos^2\theta$

$\equiv 5\sin^2\theta + 5\cos^2\theta$

$\equiv 5(\sin^2\theta + \cos^2\theta)$

$\equiv 5 \times 1$

$\equiv \underline{\underline{5}}$

(f) $2 - (\sin\theta - \cos\theta)^2 \equiv 2 - (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$

$\equiv 2 - (\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)$

$\equiv 2 - (1 - 2\sin\theta\cos\theta)$

$\equiv 2 - 1 + 2\sin\theta\cos\theta$

$\equiv 1 + 2\sin\theta\cos\theta$

$\equiv (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$

$\equiv \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$\equiv \underline{\underline{(\sin\theta + \cos\theta)^2}}$

(g) $\sin^2 x \cos^2 y - \cos^2 x \sin^2 y$

$\equiv \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$

$\equiv \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y (1 - \sin^2 x)$

$\equiv \sin^2 x - \cancel{\sin^2 x \sin^2 y} - \sin^2 y + \cancel{\sin^2 x \sin^2 y}$

$\equiv \underline{\underline{\sin^2 x - \sin^2 y}}$