Differentiation 1

Exercise 8A

1 Differentiate:

a $(1+2x)^4$ **b** $(3-2x^2)^{-5}$ **c** $(3+4x)^{\frac{1}{2}}$ **d** $(6x+x^2)^7$

e $\dfrac{1}{3+2x}$ **f** $\sqrt{7-x}$ **g** $4(2+8x)^4$ **h** $3(8-x)^{-6}$

2 Given that $y=\dfrac{1}{(4x+1)^2}$ find the value of $\dfrac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4})$.

3 Given that $y=(5-2x)^3$ find the value of $\dfrac{dy}{dx}$ at $(1, 27)$.

4 Find the value of $\dfrac{dy}{dx}$ at the point $(8, 2)$ on the curve with equation $3y^2-2y=x$.

5 Find the value of $\dfrac{dy}{dx}$ at the point $(2\frac{1}{2}, 4)$ on the curve with equation $y^{\frac{1}{2}}+y^{-\frac{1}{2}}=x$.

Exercise 8B

1 Differentiate:

a $x(1+3x)^5$ **b** $2x(1+3x^2)^3$ **c** $x^3(2x+6)^4$ **d** $3x^2(5x-1)^{-1}$

2 **a** Find the value of $\dfrac{dy}{dx}$ at the point $(1, 8)$ on the curve with equation $y=x^2(3x-1)^3$.

b Find the value of $\dfrac{dy}{dx}$ at the point $(4, 36)$ on the curve with equation $y=3x(2x+1)^{\frac{1}{2}}$.

c Find the value of $\dfrac{dy}{dx}$ at the point $(2, \frac{1}{5})$ on the curve with equation $y=(x-1)(2x+1)^{-1}$.

3 Find the points where the gradient is zero on the curve with equation $y=(x-2)^2(2x+3)$.

Exercise 8C

1 Differentiate:

a $\dfrac{5x}{x+1}$ **b** $\dfrac{2x}{3x-2}$ **c** $\dfrac{x+3}{2x+1}$ **d** $\dfrac{3x^2}{(2x-1)^2}$ **e** $\dfrac{6x}{(5x+3)^{\frac{1}{2}}}$

2 Find the value of $\dfrac{dy}{dx}$ at the point $(1, \frac{1}{4})$ on the curve with equation $y=\dfrac{x}{3x+1}$.

3 Find the value of $\dfrac{dy}{dx}$ at the point $(12, 3)$ on the curve with equation $y=\dfrac{x+3}{(2x+1)^{\frac{1}{2}}}$.

(More Exercise on Page 2)

Exercise 8D

1 Differentiate:

a e^{2x} **b** e^{-6x} **c** e^{x+3} **d** $4e^{3x^2}$ **e** $9e^{3-x}$

f xe^{2x} **g** $(x^2+3)e^{-x}$ **h** $(3x-5)e^{x^2}$ **i** $2x^4e^{1+x}$ **j** $(9x-1)e^{3x}$

k $\dfrac{x}{e^{2x}}$ **l** $\dfrac{e^{x^2}}{x}$ **m** $\dfrac{e^x}{x+1}$ **n** $\dfrac{e^{-2x}}{\sqrt{x+1}}$

2 Find the value of $\dfrac{dy}{dx}$ at the point $\left(1, \dfrac{1}{e}\right)$ on the curve with equation $y = xe^{-x}$.

3 Find the value of $\dfrac{dy}{dx}$ at the point (0, 3) on the curve with equation $y = (2x+3)e^{2x}$.

4 Find the equation of the tangent to the curve $y = xe^{2x}$ at the point $(\frac{1}{2}, \frac{1}{2}e)$.

5 Find the equation of the tangent to the curve $y = \dfrac{e^{\frac{x}{3}}}{x}$ at the point $(3, \frac{1}{3}e)$.

6 Find the coordinates of the turning points on the curve $y = x^2e^{-x}$, and determine whether these points are maximum or minimum points.

7 Given that $y = \dfrac{e^{3x}}{x}$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$, simplifying your answers.
Use these answers to find the coordinates of the turning point on the curve with equation $y = \dfrac{e^{3x}}{x}$, $x > 0$, and determine the nature of this turning point.

Exercise 8E

1 Find the function $f'(x)$ where $f(x)$ is

a $\ln(x+1)$ **b** $\ln 2x$ **c** $\ln 3x$ **d** $\ln(5x-4)$

e $3\ln x$ **f** $4\ln 2x$ **g** $5\ln(x+4)$ **h** $x\ln x$

i $\dfrac{\ln x}{x+1}$ **j** $\ln(x^2-5)$ **k** $(3+x)\ln x$ **l** $e^x \ln x$

Exercise 8F

1 Differentiate:

a $y = \sin 5x$ **b** $y = 2\sin\frac{1}{2}x$ **c** $y = 3\sin^2 x$ **d** $y = \sin(2x+1)$

e $y = \sin 8x$ **f** $y = 6\sin\frac{2}{3}x$ **g** $y = \sin^3 x$ **h** $y = \sin^5 x$

Exercise 8G

1 Differentiate:

a $y = 2\cos x$ **b** $y = \cos^5 x$ **c** $y = 6\cos\frac{5}{6}x$ **d** $y = 4\cos(3x+2)$

e $y = \cos 4x$ **f** $y = 3\cos^2 x$ **g** $y = 4\cos\frac{1}{2}x$ **h** $y = 3\cos 2x$

Exercise 8H

1 Differentiate:

a $y = \tan 3x$ **b** $y = 4\tan^3 x$ **c** $y = \tan(x-1)$ **d** $y = x^2 \tan \frac{1}{2}x + \tan(x - \frac{1}{2})$

Exercise 8I

1 Differentiate

a $\cot 4x$ **b** $\sec 5x$ **c** $\operatorname{cosec} 4x$ **d** $\sec^2 3x$

e $x \cot 3x$ **f** $\dfrac{\sec^2 x}{x}$ **g** $\operatorname{cosec}^3 2x$ **h** $\cot^2(2x-1)$

Exercise 8J

1 Find the function $f'(x)$ where $f(x)$ is

a $\sin 3x$ **b** $\cos 4x$ **c** $\tan 5x$ **d** $\sec 7x$

e $\operatorname{cosec} 2x$ **f** $\cot 3x$ **g** $\sin \dfrac{2x}{5}$ **h** $\cos \dfrac{3x}{7}$

i $\tan \dfrac{2x}{5}$ **j** $\operatorname{cosec} \dfrac{x}{2}$ **k** $\cot \frac{1}{3}x$ **l** $\sec \dfrac{3x}{2}$

2 Find the function $f'(x)$ where $f(x)$ is

a $\sin^2 x$ **b** $\cos^3 x$ **c** $\tan^4 x$ **d** $(\sec x)^{\frac{1}{2}}$

e $\sqrt{\cot x}$ **f** $\operatorname{cosec}^2 x$ **g** $\sin^3 x$ **h** $\cos^4 x$

i $\tan^2 x$ **j** $\sec^3 x$ **k** $\cot^3 x$ **l** $\operatorname{cosec}^4 x$

3 Find the function $f'(x)$ where $f(x)$ is

a $x \cos x$ **b** $x^2 \sec 3x$ **c** $\dfrac{\tan 2x}{x}$ **d** $\sin^3 x \cos x$

e $\dfrac{x^2}{\tan x}$ **f** $\dfrac{1 + \sin x}{\cos x}$ **g** $e^{2x} \cos x$ **h** $e^x \sec 3x$

i $\dfrac{\sin 3x}{e^x}$ **j** $e^x \sin^2 x$ **k** $\dfrac{\ln x}{\tan x}$ **l** $\dfrac{e^{\sin x}}{\cos x}$

Mixed exercise 8K

1 Differentiate with respect to x:

a $\ln x^2$ **b** $x^2 \sin 3x$ *E*

2 Given that

$$f(x) \equiv 3 - \frac{x^2}{4} + \ln \frac{x}{2}, \quad x > 0$$

find $f'(x)$. *E*

3 Given that $2y = x - \sin x \cos x$, show that $\dfrac{dy}{dx} = \sin^2 x$. *E*

4 Differentiate, with respect to x,

a $\frac{\sin x}{x}$, $x > 0$ **b** $\ln \frac{1}{x^2 + 9}$ **E**

5 Use the derivatives of $\sin x$ and $\cos x$ to prove that the derivative of $\tan x$ is $\sec^2 x$. **E**

6 $f(x) = \frac{x}{x^2 + 2}$, $x \in \mathbb{R}$

Find the set of values of x for which $f'(x) < 0$. **E**

7 The function f is defined for positive real values of x by

$$f(x) = 12 \ln x - x^{\frac{3}{2}}$$

Write down the set of values of x for which $f(x)$ is an increasing function of x. **E**

8 Given that $y = \cos 2x + \sin x$, $0 < x < 2\pi$, and x is in radians, find, to 2 decimal places, the values of x for which $\frac{dy}{dx} = 0$. **E**

9 The maximum point on the curve with equation $y = x\sqrt{\sin x}$, $0 < x < \pi$, is the point A. Show that the x-coordinate of point A satisfies the equation $2 \tan x + x = 0$. **E**

10 $f(x) = e^{0.5x} - x^2$, $x \in \mathbb{R}$

a Find $f'(x)$.

b By evaluating $f'(6)$ and $f'(7)$, show that the curve with equation $y = f(x)$ has a stationary point at $x = p$, where $6 < p < 7$. **E**

11 $f(x) \equiv e^{2x} \sin 2x$, $0 \leqslant x \leqslant \pi$

a Use calculus to find the coordinates of the turning points on the graph of $y = f(x)$.

b Show that $f''(x) = 8e^{2x} \cos 2x$.

c Hence, or otherwise, determine which turning point is a maximum and which is a minimum. **E**

12 The curve **C** has equation $y = 2e^x + 3x^2 + 2$. The point A with coordinates (0, 4) lies on **C**. Find the equation of the tangent to **C** at A. **E**

13 The curve **C** has equation $y = f(x)$, where

$$f(x) = 3 \ln x + \frac{1}{x}, \ x > 0$$

The point P is a stationary point on C.

a Calculate the x-coordinate of P.

The point Q on **C** has x-coordinate 1.

b Find an equation for the normal to **C** at Q. **E**

14 Differentiate $e^{2x} \cos x$ with respect to x.
The curve **C** has equation $y = e^{2x} \cos x$.

a Show that the turning points on **C** occur when $\tan x = 2$.

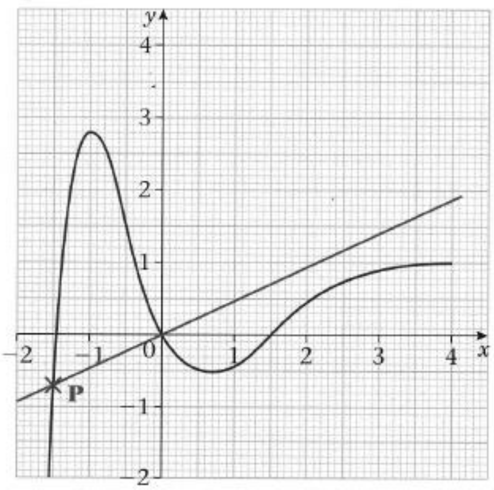
b Find an equation of the tangent to **C** at the point where $x = 0$. **E**

15 Given that $x = y^2 \ln y$, $y > 0$,

a find $\dfrac{dx}{dy}$

b use your answer to part **a** to find in terms of e, the value of $\dfrac{dy}{dx}$ at $y = e$. *E*

16



The figure shows part of the curve **C** with equation $y = f(x)$, where $f(x) = (x^3 - 2x)e^{-x}$

a Find $f'(x)$.

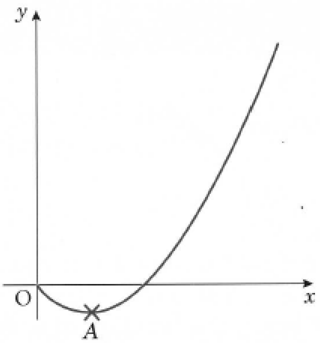
The normal to **C** at the origin O intersects **C** at a point P, as shown in the figure.

b Show that the x-coordinate of P is the solution of the equation

$$2x^2 = e^x + 4.$$

E

17



The diagram shows part of the curve with equation $y = f(x)$ where

$$f(x) = x(1 + x)\ln x \quad \{x > 0\}$$

The point A is the minimum point of the curve.

a Find $f'(x)$.

b Hence show that the x-coordinate of A is the solution of the equation $x = g(x)$, where

$$g(x) = e^{-\frac{1+x}{1+2x}}$$

E