

Trigonometry - Answers (Year 12)

Exercise A

1. (a) $\sin \theta = -1, 0 < \theta \leq 360^\circ$

$$\theta = \sin^{-1}(-1)$$

$$\theta = -90^\circ \text{ (calculator)}$$

1st Ans 2nd Ans

$$\theta = -90^\circ, (180 - -90)$$

$$= -90^\circ, \boxed{270^\circ}$$

$$\downarrow +360^\circ$$

$$\boxed{270^\circ}$$

$$\boxed{\theta = 270^\circ}$$

(b) $\tan \theta = \sqrt{3}, 0 < \theta \leq 360^\circ$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = 60^\circ \text{ (calculator)}$$

1st Ans 2nd Ans

$$\theta = \boxed{60^\circ}, 180 + 60^\circ$$

$$= \boxed{240^\circ}$$

$$\boxed{\theta = 60^\circ \text{ or } 240^\circ}$$

(c) $\cos \theta = \frac{1}{2}, 0 < \theta \leq 360^\circ$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 60^\circ \text{ (calculator)}$$

1st Ans 2nd Ans

$$\theta = \boxed{60^\circ}, 360 - 60^\circ$$

$$= \boxed{300^\circ}$$

$$\boxed{\theta = 60^\circ \text{ or } 300^\circ}$$

(d) $\sin \theta = \sin 15^\circ$

$$\theta = \sin^{-1}(\sin 15^\circ)$$

$$\theta = 15^\circ$$

Carry on as in Q1(a).

Final Answers:

$$\boxed{\theta = 15^\circ \text{ or } 165^\circ}$$

(e) $\cos \theta = -\cos 40^\circ$

$$\theta = \cos^{-1}(-\cos 40^\circ)$$

$$= 140^\circ$$

Continue the usual steps. Final Answers:

$$\theta = \boxed{140^\circ \text{ or } 220^\circ}$$

(f)

$$\theta = \boxed{135^\circ \text{ or } 315^\circ}$$

(g)

$$\theta = \boxed{90^\circ \text{ or } 270^\circ}$$

(h)

$$\theta = \boxed{230^\circ \text{ or } 310^\circ}$$

(i) $7 \sin \theta = 5$

$$\sin \theta = \frac{5}{7}$$

Continue...

$\theta = 45.6^\circ$ or 134.4°

(m)

$\theta = 131.8^\circ$ or 228.2°

(j)

$\theta = 135^\circ$ or 225°

(n)

$(\sin \theta - 1)(5 \cos \theta + 3) = 0$

$\sin \theta - 1 = 0$ or $5 \cos \theta + 3 = 0$

$\sin \theta = 1$ or $\cos \theta = -3/5$

Solve each of these following the usual steps and finally give all the possible answers together.

Final Answers:

$\theta = 90^\circ, 126.9^\circ$ or 233.1°

(k) $\sqrt{3} \sin \theta = \cos \theta$

$(\div \cos \theta)$ $(\div \cos \theta)$

$\frac{\sqrt{3} \sin \theta}{\cos \theta} = 1$

$\sqrt{3} \tan \theta = 1$

$\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = 30^\circ$ (calculator)

Continue...

Final Answers:

$\theta = 30^\circ$ or 210°

(l) $\sin \theta + \cos \theta = 0$

$\sin \theta = -\cos \theta$

$(\div \cos \theta)$ $(\div \cos \theta)$

$\frac{\sin \theta}{\cos \theta} = -1$

$\tan \theta = -1$

Continue...

$\theta = 135^\circ$ or 315°

(o) $\tan \theta = (\tan \theta)(2 + 3 \sin \theta)$

$\tan \theta - \tan \theta(2 + 3 \sin \theta) = 0$

$\tan \theta [1 - (2 + 3 \sin \theta)] = 0$

$\tan \theta (1 - 2 - 3 \sin \theta) = 0$

$\tan \theta (-1 - 3 \sin \theta) = 0$

$\tan \theta (-1 - 3 \sin \theta) = 0$

$\tan \theta = 0$ or $-1 - 3 \sin \theta = 0$

$\tan \theta = 0$ or $\sin \theta = -1/3$

Continue...

Final Answers:

$\theta = 180^\circ, 199.5^\circ, 340.5^\circ$
or 360°

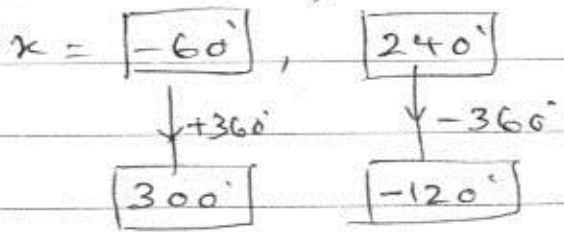
2.

(a) $\sin x = -\frac{\sqrt{3}}{2}, -180 \leq x \leq 540$

$x = \sin^{-1}(-\sqrt{3}/2)$
 $x = -60$ (calculator)

1st Ans 2nd Ans

$x = -60, (180 - -60)$

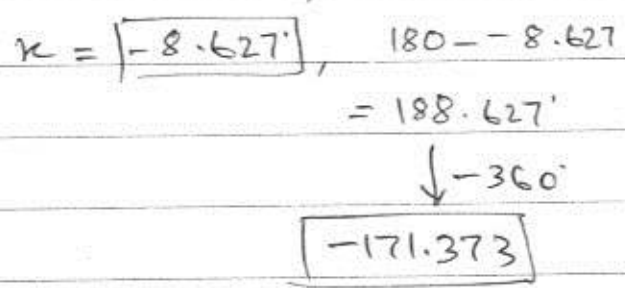


$x = -60, -120, 240$ or 300

(b) $2 \sin x = -0.3, -180 \leq x \leq 180$

$\sin x = \frac{-0.3}{2}$
 $x = \sin^{-1}(-0.3/2)$
 $x = -8.627$ (calculator)

1st Ans 2nd Ans



$x = -8.63$ or -171 (3 s.f.)

(c)

$x = -144$ or 144

(d)

$x = -327$ or -32.9

(e)

$x = 150, 330, 510$ or 690

(f)

$x = 251$ or 431

3

(a) $\sin 4\theta = 0, 0 \leq \theta \leq 360$

$4\theta = \sin^{-1}(0)$

$4\theta = 0$

1st Ans 2nd Ans

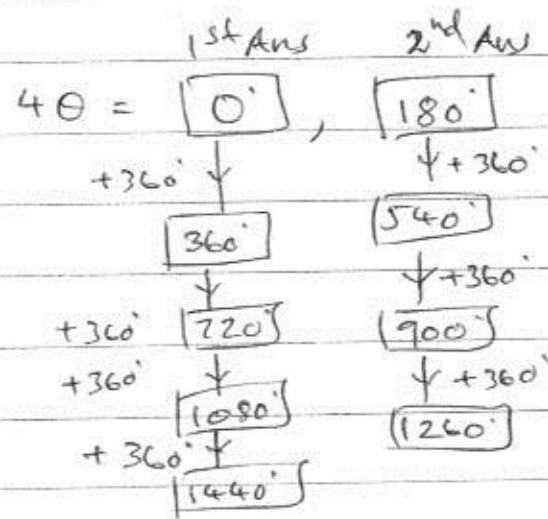
$4\theta = \boxed{0}, 180 - 0$
 $= \boxed{180}$

Range of 4θ :

$0 \leq \theta \leq 360$

(x4)

$0 \leq 4\theta \leq 1440$



$$4\theta = 0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ, 900^\circ, 1080^\circ, 1260^\circ \text{ or } 1440^\circ$$

$$\therefore \theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ \text{ or } 360^\circ$$

(c)

$$\theta = 22.5^\circ, 112.5^\circ, 202.5^\circ \text{ or } 292.5^\circ$$

(d)

$$\theta = 30^\circ, 150^\circ, 210^\circ \text{ or } 330^\circ$$

(b) $\cos 3\theta = -1, 0 \leq \theta \leq 360^\circ$

$$3\theta = \cos^{-1}(-1)$$

$$3\theta = 180^\circ$$

1st Ans 2nd Ans

$$3\theta = 180^\circ, 360 - 180^\circ = 180^\circ$$

Range of 3θ :

$$0 \leq \theta \leq 360^\circ$$

(x3)

$$0 \leq 3\theta \leq 1080^\circ$$

$$3\theta = 180^\circ$$

$$\downarrow +360^\circ$$

$$540^\circ$$

$$\downarrow +360^\circ$$

$$900^\circ$$

$$\therefore 3\theta = 180^\circ, 540^\circ \text{ or } 900^\circ$$

$$\theta = 60^\circ, 180^\circ \text{ or } 300^\circ$$

(e)

$$\tan \frac{1}{2}\theta = -\frac{1}{\sqrt{3}}$$

$$\frac{1}{2}\theta = \tan^{-1}(-\frac{1}{\sqrt{3}})$$

$$\frac{1}{2}\theta = -30^\circ$$

1st Ans 2nd Ans

$$\frac{1}{2}\theta = -30^\circ, 180 + (-30^\circ) = 150^\circ$$

Range of $\frac{1}{2}\theta$:

$$0 \leq \theta \leq 360^\circ$$

(x $\frac{1}{2}$)

$$0 \leq \frac{1}{2}\theta \leq 180^\circ$$

$$\frac{1}{2}\theta = -30^\circ, 150^\circ$$

$$\therefore \frac{1}{2}\theta = 150^\circ$$

$$\theta = 300^\circ$$

(f) $\sin(-\theta) = \frac{1}{\sqrt{2}}$

$$-\theta = \sin^{-1}(\frac{1}{\sqrt{2}})$$

$$-\theta = 45^\circ$$

$$-\theta = \begin{matrix} \text{1st Ans} & \text{2nd Ans} \\ 45^\circ & 180 - 45^\circ \\ & = 135^\circ \end{matrix}$$

Range of $-\theta$:

$$0 \leq \theta \leq 360^\circ$$

(x-1)

$$0 \geq -\theta \geq -360^\circ$$

That is,

$$-360^\circ \leq -\theta \leq 0$$

$$-\theta = \begin{matrix} \text{1st Ans} & \text{2nd Ans} \\ 45^\circ & 135^\circ \\ \downarrow -360^\circ & \downarrow -360^\circ \\ \boxed{-315^\circ} & \boxed{-225^\circ} \end{matrix}$$

$$-\theta = -315^\circ \text{ or } -225^\circ$$

$$\therefore \theta = 315^\circ \text{ or } 225^\circ$$

$$0 \geq -\theta \geq -360^\circ$$

(+45°)

$$45^\circ \geq -\theta + 45^\circ \geq -315^\circ$$

That is,

$$-315^\circ \leq 45^\circ - \theta \leq 45^\circ$$

$$45^\circ - \theta = \begin{matrix} \text{1st Ans} & \text{2nd Ans} \\ \boxed{-45^\circ} & 135^\circ \\ & \downarrow -360^\circ \end{matrix}$$

$$\boxed{-225^\circ}$$

$$45^\circ - \theta = -45^\circ \text{ or } -225^\circ$$

$$-\theta = -90^\circ \text{ or } -270^\circ$$

$$\therefore \theta = 90^\circ \text{ or } 270^\circ$$

(h)

$$\theta = 50^\circ \text{ or } 170^\circ$$

(i)

$$\theta = 165^\circ \text{ or } 345^\circ$$

(j)

$$\theta = 65^\circ \text{ or } 245^\circ$$

(g) $\tan(45^\circ - \theta) = -1$

$$45^\circ - \theta = \tan^{-1}(-1)$$

$$45^\circ - \theta = -45^\circ$$

$$\begin{matrix} \text{1st Ans} & \text{2nd Ans} \end{matrix}$$

$$45^\circ - \theta = -45^\circ, \quad 180 + -45^\circ = 135^\circ$$

Range of $45^\circ - \theta$:

$$0 \leq \theta \leq 360^\circ$$

(x-1)

$$0 \geq -\theta \geq -360^\circ$$