Answers: Gravitational Fields 1

Exercise A

1.

$g = \frac{F}{m}$ [1]

$F = ma$ therefore in base units $F = kg\,m\,s^{-2}$ [1]

m in base units: kg

g therefore: $kg\,m\,s^{-2}\,kg^{-1} = m\,s^{-2}$ [1]

2.

On Earth: $F = mg = 75 \times 9.81 = 740\,N$ and

On Mars $F = mg = 75 \times 3.7 = 280\,N$ [1]

Change = 740 − 280 = 460 N [1]

Exercise B

1 Since $F = \frac{GMm}{r^2}$ and $y = mx + c$ [1]

F on the y-axis and $\frac{1}{r^2}$ on the x-axis [1]

Gradient = GMm ($F = GMm\frac{1}{r^2} \rightarrow y = mx$) [1]

2 $F = \frac{GMm}{r^2}$

$M = M_E$ and $m = m_S$ [1]

and $r = R_E + h$. [1]

Therefore $F = \frac{GM_E m_s}{(R_E + h)^2}$

3 a Since $F \propto M$ if M doubles F doubles. [1]

b Since $F \propto Mm$ if M doubles and m doubles, F quadruples. [1]

c Since $F \propto \frac{1}{r^2}$ if r halves F quadruples. [1]

d Since $F \propto M$ if M doubles F doubles and since $F \propto \frac{1}{r^2}$ if r decreases by a factor of four F increases by a factor of 16. [1]

Therefore in total F increases by a factor of 32. [1]

4 a $F = \frac{GMm}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 1.67 \times 10^{-27}}{(1.0 \times 10^{-14})^2}$ [1]

$F = 1.9 \times 10^{-36}$ N [1]

b $F = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 65 \times 70}{(1.5)^2}$ [1]

$F = 1.3 \times 10^{-7}$ N [1]

c $F = \frac{GMm}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 5.68 \times 10^{26}}{(1400 \times 10^9)^2}$ [1]

$F = 3.8 \times 10^{22}$ N [1]

5 $F = \frac{GMm}{r^2}$ therefore $m = \frac{Fr^2}{GM}$ [1]

$m = \frac{2.03 \times 10^{20} \times (380 \times 10^6)^2}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}$ [1]

$m = 7.36 \times 10^{22}$ kg [1]

6 Net force on probe $F = \frac{GM_E m_{probe}}{r_{E\ to\ probe}^2} - \frac{GM_M m_{probe}}{r_{M\ to\ probe}^2}$ [1]

$F = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 120}{(190 \times 10^6)^2}$

$- \frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22} \times 120}{(190 \times 10^6)^2}$ [1]

$F = 1.31$ N (towards the Earth) [1]

Exercise C

1 Gravitational field strength is a vector quantity. Since gravitational fields are always attractive, the gravitational fields of the Earth and Moon are in opposite directions.

2 At position Z the net gravitational field strength is zero, therefore:

$-\frac{GM_{Earth}}{r_{Earth\ to\ Z}^2} = -\frac{GM_{Moon}}{r_{Moon\ to\ Z}^2}$

$\sqrt{\frac{M_{Earth}}{M_{Moon}}} = \frac{r_{Earth\ to\ Z}}{r_{Moon\ to\ Z}}$

$\sqrt{\frac{5.97 \times 10^{24}}{7.35 \times 10^{22}}} = 9.01 = \frac{r_{Earth\ to\ Z}}{r_{Moon\ to\ Z}}$

Therefore the distance must be 9 times greater from the Earth than to the Moon.

$\frac{380000 \times 9}{10}$ = 342 000 km from the Earth (38 000 km from Moon).

3 The mass of the Moon is much smaller than that of the Earth, therefore its gravitational field strength is much weaker at the same distance from its centre of mass.

In order to send a spacecraft from the Earth to the Moon, work must be done up to the point where the Moon's gravitational field becomes greater than the Earth's (and so attracts the spacecraft). This point is much closer to the Moon than the Earth.

Exercise D

1 Since diameter = 1.39 million km, radius = 695×10^6 m [1]

$g = -\frac{GM}{r^2} = -\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(695 \times 10^6)^2}$ [1]

$g = -275\,\text{N kg}^{-1}$ [1]

2 $g = -\frac{GM}{r^2} = -\frac{6.67 \times 10^{-11} \times 2.60 \times 10^{23}}{(1.2 \times 10^8)^2}$ [1]

$g = -1.2 \times 10^{-3}\,\text{N kg}^{-1}$ [1]

3 a Since $g \propto M$, if M halves, g doubles. [1]

b Since $g \propto \frac{1}{r^2}$, if r increases by a factor of three, g decreases by a factor of 9 (3^2). [1]

c Since $g \propto M$, if M decreases by a factor of four, g decreases by a factor of four, and since $g \propto \frac{1}{r^2}$, if r decreases by a factor of two, g increases by a factor of four. [1]. Therefore overall there is no change in g. [1]

4 $g \propto \frac{1}{r^2}$ however r is measured from the centre of mass. [1]

Moving from 100 m above to surface to 200 m above the surface does not double r (6400.1 km to 6400.2 km). [1]

5 At the poles: $g = -\frac{GM}{r^2} = -\frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6371\times10^3)^2}$

$= 9.81\,\text{N kg}^{-1}$ [1]

At the equator: $g = -\frac{GM}{r^2} = -\frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6378\times10^3)^2}$

$= 9.79\,\text{N kg}^{-1}$ [1]

% change = 0.2 % [1]

6 $g = -\frac{GM}{r^2}$ therefore $r = \sqrt{\frac{GM}{g}}$ [1]

$r = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6.67\times10^{-11}\times6.42\times10^{23}}{3.72}}$ [1]

$r = 3.4\times10^6\,\text{m}$ [1]

7 $g = \frac{GM}{r^2}$ therefore $M = \frac{gr^2}{G}$ [1]

$M = \frac{8.77\times(6.09\times10^6)^2}{6.67\times10^{-11}}$ [1]

$= 4.88\times10^{24}\,\text{kg}$ [1]

Exercise E

1 See Figures 2 and 3 in the main content pages.

Diagram should include two planets at different distances from the star. [1]

Diagram showing two elliptical orbits with the Sun at a focus [1] Kepler's first law.

Diagram should also show that a line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time (for one or both orbits). [1] Kepler's second law.

The diagram should also show (in words) $T^2 \alpha\, r^3$ [1]

2 $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ gives $T = \sqrt{\left(\frac{4\pi^2}{GM}\right)r^3}$ [1]

$T = \sqrt{\left(\frac{4\pi^2}{6.67\times10^{-11}\times1.99\times10^{30}}\right)(1400\times10^9)^3}$ [1]

$T = 900\times10^6\,\text{s}$ [1]

3 a Since $T^2 \alpha\, r^3$ if r doubles T^2 increases by a factor of 8 (2^3)[1], therefore T increase by a factor of $\sqrt{8}$ (2.8) [1]

b Since $T^2 \alpha\, r^3$ if r increases by a factor of three T^2 increases by a factor of 27 (3^3)[1], therefore T increases by a factor of $\sqrt{27}$ (5.2) [1]

c Since $T^2 \alpha\, r^3$ if r decreases by a factor of nine T^2 decreases by a factor of 729 (9^3)[1], therefore T decreases by a factor of $\sqrt{729}$ (27) [1]

4 Kepler's third law: $\frac{T^2}{r^3} = k$ [1]

Calculation of k for each moon [1] (any units e.g.)

Moon	r / × 10^3 km	T / days	k × 10^{-17} days^2 km^{-3}
Io	420	1.8	4.37
Europa	670	3.6	4.31
Ganymede	1070	7.2	4.23
Callisto	1890	16.7	4.13

k is approximately constant. (<6% variation). [1]

5 Graph of T^2 against r^3, axis labelled correctly with quantities and units, points plotted correctly and line of best fit drawn. [3]

Gradient $= \frac{4\pi^2}{GM}$ [1]

Therefore $M = \frac{4\pi^2}{G \times \text{gradient}} = 1.9 \times 10^{27}\,\text{kg}$ [1]

Exercise F

1 Kepler's third law states $T^2 \alpha\, r^3$ [1], therefore if r reduces, T also reduces. [1]

2 Only one force [1]

Gravitational attraction towards the centre of mass of the Earth [1]

3 a Nine times in one day, therefore the period $= \frac{24}{9} = 2.7\,\text{hours} = 9600\,\text{s}$ [1]

b $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ therefore: $r = \sqrt[3]{\frac{T^2}{\left(\frac{4\pi^2}{GM}\right)}}$ [1].

$$r = \sqrt[3]{\frac{9600^2}{\left(\frac{4\pi^2}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}\right)}} = 9.8 \times 10^6\,\text{m} \quad [1]$$

c $F = \frac{mv^2}{r}$ [1] $F = \frac{180 \times 6400^2}{9.8 \times 10^6} = 750\,\text{N}$ [1]

d $a = \frac{v^2}{r}$ [1] $a = \frac{v^2}{r} = \frac{6400^2}{9.8 \times 10^6} = 4.2\,\text{m}\,\text{s}^{-2}$ [1]

4 The smallest value for $r = 6370\,\text{km}$ (radius of the Earth). [1]

$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ [1]

$$T = \sqrt{\left(\frac{4\pi^2}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}\right) \times \left(6370 \times 10^3\right)^3} \quad [1]$$

$T = 5060\,\text{s} = 85\,\text{mins}$ [1]

5 $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ and $r = 6370 + 5000\,\text{km} = 11\,370\,\text{km}$ [1]

$T = \sqrt{\left(\frac{4\pi^2}{6.67\times10^{-11}\times5.97\times10^{24}}\right)\times(11370\times10^3)^3}$ [1]

$T = 12\,100\,\text{s}$ [1]

$v = \frac{2\pi r}{T} = \frac{2\times\pi\times11370\times10^3}{12100} = 5900\,\text{m s}^{-1}$ [1]

Exercise G

1 $V_g = -\frac{GM}{r}$ [1]

$V_g = -\frac{6.67\times10^{-11}\times7.10\times10^{21}}{3.4\times10^6} = -140\,\text{kJ kg}^{-1}$ [1]

2 a Since $V_g \propto M$, if M doubles, V_g doubles. [1]

b Since $V_g \propto \frac{1}{r}$, if r decreases by a factor of four, V_g increases by a factor of 4. [1]

c Since $V_g \propto M$, if M increases by a factor of three, V_g increases by a factor of three and since $V_g \propto \frac{1}{r}$, if r doubles, V_g decreases by a factor of two. [1]

Resulting in a total increase by a factor of 1.5. [1]

3 Since $V_g = -\frac{GM}{r}$, as M is constant [1] and r is constant (at a fixed height) [1] V_g must be constant.

4 $V_g = -\frac{GM}{r}$ and radius = 695 Mm [1]

$V_g = -\frac{6.67\times10^{-11}\times1.99\times10^{30}}{695\times10^6}$

$= -1.91\times10^{11}\,\text{J kg}^{-1} = -1.9\times10^{11}\,\text{J kg}^{-1}$ [1]

5 Graph of V_g against $\frac{1}{r}$, axis labelled correctly with quantities and units, points plotted correctly and line of best fit drawn. [4]

Gradient = $-GM$ [1]

Therefore $M = -\frac{\text{Gradient}}{G} = 5.94\times10^{24}\,\text{kg}$ [1]

Exercise H

1 a 235 J

2 a 2.0 MJ kg^{-1}

b i −61 MJ kg^{-1}

ii 2.2×10^9 J

3 a i −250 J

ii −200 J

iii −200 J

b i 50 J

ii 0

4 b 5 N kg^{-1}

c 25 MJ

Exercise I

1 For there to be a change in gravitational potential energy a fixed mass must experience a change in gravitational potential. [1]

At a fixed height in a uniform gravitational field the gravitational potential is constant. Only a change in vertical height will results in a change in gravitational potential and so a change in gravitational potential energy. [1]

2 a $E = mV_g = 40 \times -32\times10^6 = -1.3\times10^9$ J [1]

b $E = mV_g = 7.4\times10^{-6} \times -32\times10^6 = -240$ J [1]

c $E = mV_g = 1.67\times10^{-27} \times -32\times10^6$

$= -5.3\times10^{-20}$ J [1]

3 $v = \sqrt{\frac{2GM}{r}}$ [1]

$v = \sqrt{\frac{2\times6.67\times10^{-11}\times7.35\times10^{22}}{1740\times10^3}}$ [1]

$v = 2400\,\text{m s}^{-1}$ [1]

4 On the surface: $V_g = -\frac{GM}{r} = -\frac{6.67\times10^{-11}\times5.97\times10^{24}}{6370\times10^3}$

$= -62.5\,\text{MJ kg}^{-1}$ [1]

At a height of 50 000 km the surface: $V_g = -\frac{GM}{r}$

$= -\frac{6.67\times10^{-11}\times5.97\times10^{24}}{56370\times10^3} = -7.06\,\text{MJ kg}^{-1}$ [1]

$\Delta V_g = 55.4\,\text{MJ kg}^{-1}$ [1]

$\Delta E = m\Delta V_g = 300\times55.4\times10^6 = 1.66\times10^{10}$ [1]

5 As the comet accelerates towards the Sun it loses gravitational potential energy and gains kinetic energy. Therefore: $\frac{1}{2}mv^2 = \frac{GMm}{r}$ [1]

On impact $v = \sqrt{\frac{2GM}{r}}$ [1]

$v = \sqrt{\frac{2\times6.67\times10^{-11}\times1.99\times10^{30}}{696\times10^6}} = 620\,\text{km s}^{-1}$ [1]