Gravitational Fields 1

Exercise A

1.
Use the defining equation for gravitational field strength to show that alternative units of g are m s^{-2}. (*3 marks*)

2.
The gravitational field strength g on the surface of Mars is 3.7 N kg^{-1}. Calculate the difference in the gravitational force experienced by an astronaut of mass of 75 kg on the surface of Mars compared with the gravitational force experienced by the same astronaut on the surface of the Earth. (*2 marks*)

Exercise B

1 Explain why the gradient of the graph of F against $\frac{1}{r^2}$ is equal to GMm. (*3 marks*)

2 Show that the gravitational force on a satellite of mass m_s at a height h above the surface of the Earth is given by

$$F = \frac{GM_E m_s}{(R_E + h)^2}$$

where M_E and R_E are the mass and radius of the Earth respectively. (*2 marks*)

3 Describe what happens to the gravitational force between two objects A and B when:
a the mass of A doubles;
b both masses A and B double;
c the distance between A and B halves;
d the mass of B doubles and the distance between A and B decreases by a factor of four. (*5 marks*)

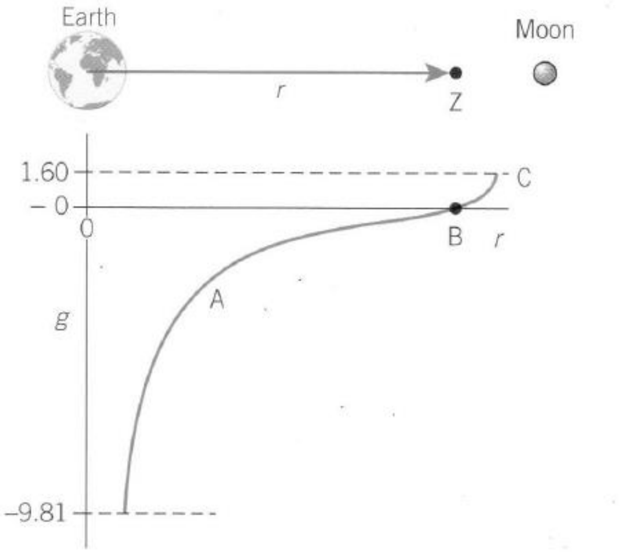
4 Calculate the gravitational force between:
a two protons of mass 1.67×10^{-27} kg separated by a distance of 1.0×10^{-14} m;
b two students of mass 65 kg and 70 kg standing 1.5 m apart;
c Saturn and the Sun (mass of Sun = 1.99×10^{30} kg, mass of Saturn = 5.68×10^{26} kg, average separation from the Sun to Saturn = 1400 million km). (*6 marks*)

5 The mean distance from the centre of the Earth to the centre of the Moon is about 380 000 km. The gravitational force between the Earth and the Moon is 2.03×10^{20} N. The mass of the Earth is 5.97×10^{24} kg. Calculate the mass of the Moon. (*3 marks*)

6 Use the information in question 5 to determine the resultant force on a probe of mass 120 kg when it is halfway between the Earth and the Moon. (*3 marks*)

Exercise C

Use Figure 3 given below to answer the questions in this exercise.



▲ **Figure 3** *A graph to show the resultant gravitational field strength on the journey from the Earth to the Moon*

1. Explain why the resultant gravitational field strength changes from a negative value near the Earth to a positive value near the Moon.
2. The distance between the centres of the Earth and the Moon is 3.8×10^{8} m. Calculate the distance from the centre of mass of the Earth to position Z in Figure 3.
3. Explain why position Z is much closer to the Moon than the Earth and therefore explain why it requires much more energy to send a spacecraft to the Moon than for it to return from the Moon to the Earth.

Exercise D

1. The Sun has a mass of 1.99×10^{30} kg and a diameter of 1.39 million km. Calculate the gravitational field strength at its surface. *(3 marks)*
2. Calculate the gravitational field strength 1.2×10^{8} m from a point mass of 2.6×10^{23} kg. *(2 marks)*
3. State the effect on the gravitational field strength at a point in a radial field around a point mass when:
   a. the mass of the point mass creating the field is halved;
   b. the distance from the point mass increases by a factor of three;
   c. the mass of the point mass decreases by a factor of four and the distance from the point mass halves. *(4 marks)*
4. Explain why, when moving an object from a height of 100 m to a height of 200 m above the surface of the Earth, the gravitational field strength does not decrease by a factor of four. *(2 marks)*
5. The Earth is not perfectly spherical. The radius at the equator is 6378 km and at the poles is 6371 km. Calculate the percentage change in the gravitational field strength between the equator and the poles. (Mass of the Earth = 5.97×10^{24} kg.) *(3 marks)*
6. Mars has a mass of 6.42×10^{23} kg and a surface gravitational field strength of 3.72 N kg^{-1}. Calculate the radius of Mars. *(3 marks)*
7. The surface gravitational field strength on Venus is 8.77 N kg^{-1} and it has a radius of 6.09 Mm. Calculate the mass of Venus. *(3 marks)*

Exercise E

1 Sketch a single diagram showing the orbit of two planets around a star and use your diagram to describe Kepler's three laws of planetary motion. (4 marks)

2 Saturn is at a mean distance of 1400 million km from the Sun. Calculate the orbital period of Saturn in Earth years. The mass of the Sun is 1.99×10^{30} kg. (3 marks)

3 State the effect on the orbital period of a planet when the distance from a planet to the star:
a doubles; b increases by a factor three; c decreases by a factor of nine. (6 marks)

4 Use the data in Table 2 to confirm that the moons of Jupiter obey Kepler's third law. (3 marks)

5 Plot a graph of T^2 against r^3 for the moons shown in Table 2 and use your graph to show that the mass of Jupiter is approximately 1.9×10^{27} kg. (5 marks)

Exercise F

Data required for questions 3, 4, and 5: mass of the Earth = 5.97×10^{24} kg and radius of the Earth = 6370 km.

1 Use Kepler's third law to explain why satellites closer to the surface of the Earth take less time to orbit than those higher up. (2 marks)

2 Draw a labelled diagram to show the forces acting on a satellite in orbit. (2 marks)

3 A 180 kg satellite in a polar orbit travels at 6400 m s^{-1} circling the Earth nine times in one day. Calculate:
a its orbital period;
b the radius of its orbit;
c the gravitational force acting on the satellite;
d its centripetal acceleration. (7 marks)

4 Show that the lowest theoretical orbital period for a satellite around Earth is around 85 minutes. (4 marks)

5 Calculate the velocity of a satellite in orbit at a height of 5000 km above the surface of the Earth. (4 marks)

Exercise G

1 Calculate the gravitational potential at a point 3.4×10^6 m from a 7.10×10^{21} kg point mass. (2 marks)

2 State the effect on the gravitational potential at a point in a radial field around a point mass if:
a the mass of the point mass creating the field doubles;
b the distance from the point mass decreases by a factor of four;
c the mass of the point mass increases by a factor of three and the distance from the point mass doubles. (4 marks)

3 Explain why a satellite in a circular orbit at a fixed height does not experience a change in gravitational potential. (2 marks)

4 The Sun has mass 1.99×10^{30} kg and diameter 1.39×10^9 m. Show that the gravitational potential on its surface is about -1.9×10^{11} J kg^{-1}. (2 marks)

5 Take six values from the graph in Figure 4 and plot a graph of V_g against $\frac{1}{r}$. Use your graph to determine the mass of the Earth. (6 marks)

Exercise H

$g = 9.8\ N\ kg^{-1}$

1 a Calculate the gain of gravitational potential energy of an object of mass 12 kg when its centre of mass is raised through a height of 2.0 m.

b Demonstrate that the gravitational potential difference between the Earth's surface and a point 2.0 m above the surface is $19.6\ J\ kg^{-1}$.

2 A rocket of mass 35 kg launched from the Earth's surface gains 70 MJ of gravitational potential energy when it reaches its maximum height.

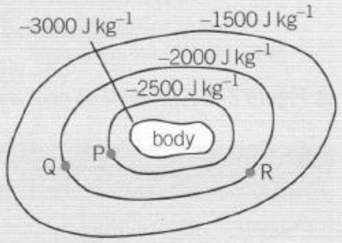
a Calculate the gravitational potential difference between the Earth's surface and the highest point reached by the rocket.

b The gravitational potential of the Earth's gravitational field at the surface of the Earth is $-63\ MJ\ kg^{-1}$. Calculate:

i the gravitational potential at the highest point reached by the rocket,

ii the work that would need to have been done by the rocket to escape from the Earth's gravitational field.

3 Figure 4 shows the equipotentials near a non-spherical object.



▲ Figure 4

a Calculate the gravitational potential energy of a 0.1 kg object at:

i P,

ii Q,

iii R.

b Calculate how much work must be done on the object to move it from:

i P to Q,

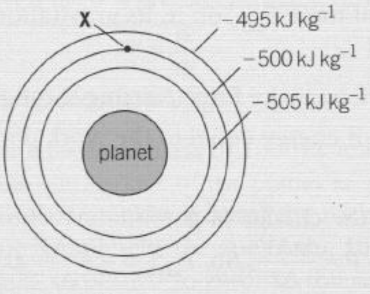
ii Q to R.

4 Figure 5 shows equipotentials at a spacing of 1.0 km near a planet. The point labelled X is on the $-500\ kJ\ kg^{-1}$ equipotential.

a Demonstrate that the potential gradient at X is $5.0\ J\ kg^{-1}\ m^{-1}$,

b Hence calculate the gravitational field strength at X.

c Calculate the work that would need to be done to remove an object of mass 50 kg from X to infinity.



▲ Figure 5

Exercise I

1 Explain why in order for there to be a change in gravitational potential energy a mass needs to move vertically in a uniform field. *(2 marks)*

2 Calculate the gravitational potential energy of the following masses at a point in a gravitational field where the gravitational potential is $-32\ MJ\ kg^{-1}$: **a** 40 kg; **b** 7.4 μg; **c** 1.67×10^{-27} kg (mass of a proton). *(3 marks)*

3 On the Apollo 14 mission to the Moon, astronaut Alan Shepard smuggled a golf club on board. He used it to strike a golf ball around 300 m on the surface. The Moon has mass 7.35×10^{22} kg and radius 1740 km. Calculate the velocity needed by the golf ball to escape from the surface of the Moon. *(3 marks)*

4 The Earth has mass 5.97×10^{24} kg and radius 6370 km. Calculate the change in gravitational potential energy required to lift 300 kg into an orbit 50 000 km above the surface of the Earth. *(4 marks)*

5 The Sun has mass 1.99×10^{30} kg and radius 6.96×10^{8} m. A very distant comet is 'caught' by the Sun's gravitational field and accelerates towards the centre of the Sun. Estimate the speed of the comet at the edge of the Sun. *(3 marks)*