Mixed Exercise - 3

1.

The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)

2.

The circle C, with centre at the point A, has equation $x^2 + y^2 - 10x + 9 = 0$.

Find

(a) the coordinates of A,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points at which C crosses the x-axis.

(2)

Given that the line l with gradient $\frac{7}{2}$ is a tangent to C, and that l touches C at the point T,

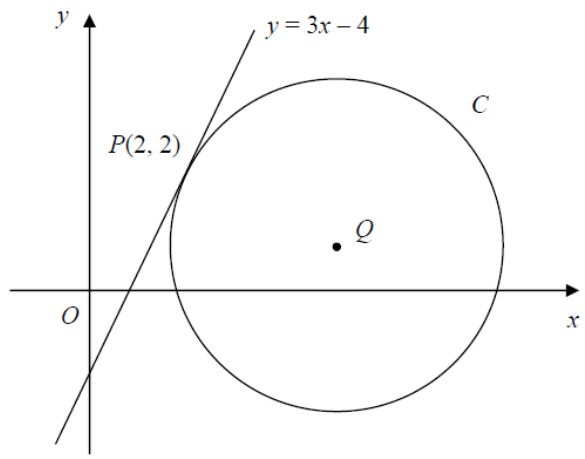
(d) find an equation of the line which passes through A and T.

(3)

Question 3 is on the next page.

3.

Figure 1



The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point P(2, 2), as shown in Figure 1.

The point Q is the centre of C.

(a) Find an equation of the straight line through P and Q.

(3)

Given that Q lies on the line $y = 1$,

(b) show that the x-coordinate of Q is 5,

(1)

(c) find an equation for C.

(4)

4.

The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x + 4y = 7$.

(a) (i) Show that $k = -2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

5.

(a) (i) Express $x^2 - 4x + 9$ in the form $(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

(b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 - 2x - 3 = 0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

6.

The quadratic equation $x^2 + (m + 4)x + (4m + 1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

---

7. Solve ,

$$2x^2 - 5x - 7 < 0$$

---

8. Sketch the following graphs showing the coordinates of the points where they intersect the coordinate axes.

(a) $y = (2 - x)(x - 4)(x - 1)$

(b) $y = (x - 1)^2 (x + 6)$

(c) $y = x^3 + 5x^2 + 4x$

9.

a Expand $(1 - 2x)^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient in the expansion.

b Use your expansion to find an approximation to $(0.98)^{10}$, stating clearly the substitution which you have used for x.

10.

If x is so small that terms of x^3 and higher can be ignored, show that:

$(2 + x)(1 - 3x)^5 \approx 2 - 29x + 165x^2$