Trigonometry – Answers (Year 13)

Exercise A

1 **a** $\operatorname{cosec}^3\theta$ **b** $2\cot^3\theta$ **c** $\frac{1}{2}\sec^2\theta$
d $\cot^2\theta$ **e** $\sec^5\theta$ **f** $\operatorname{cosec}^2\theta$
g $2\cot^{\frac{1}{2}}\theta$ **h** $\sec^3\theta$

2 **a** $\frac{5}{4}$ **b** $-\frac{1}{2}$ **c** $\pm\sqrt{3}$

3 **a** $\cos\theta$ **b** 1 **c** $\sec 2\theta$
d 1 **e** 1 **f** $\cos A$
g $\cos x$

4 **a** L.H.S. $= \cos\theta + \sin\theta\dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} = \sec\theta = \text{R.H.S.}$

b L.H.S. $= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{1}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta} \times \dfrac{1}{\cos\theta}$

$= \operatorname{cosec}\theta\sec\theta = \text{R.H.S.}$

c L.H.S. $= \dfrac{1}{\sin\theta} - \sin\theta \equiv \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

$= \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta = \text{R.H.S.}$

d L.H.S. $= 1 - \cos x + \sec x - 1 = \sec x - \cos x$

$= \dfrac{1}{\cos x} \quad \cos x = \dfrac{1-\cos^2 x}{\cos x} = \dfrac{\sin^2 x}{\cos x}$

$= \sin x \times \dfrac{\sin x}{\cos x} = \sin x\tan x = \text{R.H.S.}$

e L.H.S. $= \dfrac{\cos^2 x + (1-\sin x)^2}{(1-\sin x)\cos x}$

$= \dfrac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{(1-\sin x)\cos x}$

$= \dfrac{2 - 2\sin x}{(1-\sin x)\cos x} = \dfrac{2(1-\sin x)}{(1-\sin x)\cos x}$

$= 2\sec x = \text{R.H.S.}$

f L.H.S. $= \dfrac{\cos\theta}{1+\dfrac{1}{\tan\theta}} = \dfrac{\cos\theta}{\left(\dfrac{\tan\theta+1}{\tan\theta}\right)}$

$= \dfrac{\cos\theta\tan\theta}{\tan\theta+1} = \dfrac{\sin\theta}{1+\tan\theta} = \text{R.H.S.}$

5 **a** 45°, 315° **b** 199°, 341°
c 112°, 292° **d** 30°, 150°
e 30°, 150°, 210°, 330° **f** 36.9°, 90°, 143°, 270°
g 26.6°, 207° **h** 45°, 135°, 225°, 315°

6 **a** 90° **b** ±109°
c −164°, 16.2° **d** 41.8°, 138°
e ±45°, ±135° **f** ±60°
g −173°, −97.2°, 7.24°, 82.8°
h −152°, −36.5°, 28.4°, 143°

7 **a** π **b** $\dfrac{5\pi}{6}, \dfrac{11\pi}{6}$ **c** $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ **d** $\dfrac{\pi}{4}, \dfrac{3\pi}{4}$

Exercise B

1 **a** $\sec^2\left(\frac{1}{2}\theta\right)$ **b** $\tan^2\theta$ **c** 1
d $\tan\theta$ **e** 1 **f** 3
g $\sin\theta$ **h** 1 **i** $\cos\theta$
j 1 **k** $4\operatorname{cosec}^4 2\theta$

2 $\pm\sqrt{k-1}$

3 **a** $\frac{1}{2}$ **b** $-\dfrac{\sqrt{3}}{2}$

4 **a** $-\frac{5}{4}$ **b** $-\frac{4}{5}$ **c** $-\frac{3}{5}$

5 **a** $-\frac{7}{24}$ **b** $-\frac{25}{7}$

6 **a** L.H.S. $= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$
$= 1(\sec^2\theta + \tan^2\theta) =$ R.H.S.

b L.H.S. $\equiv (1 + \cot^2 x) - (1 - \cos^2 x)$
$\equiv \cot^2 x + \cos^2 x =$ R.H.S.

c L.H.S. $\equiv \dfrac{1}{\cos^2 A}\left(\dfrac{\cos^2 A}{\sin^2 A} - \cos^2 A\right) = \dfrac{1}{\sin^2 A} - 1$
$= \operatorname{cosec}^2 A - 1 = \cot^2 A =$ R.H.S.

d R.H.S. $\equiv \tan^2\theta \times \cos^2\theta \equiv \dfrac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta \equiv \sin^2\theta$
$\equiv 1 - \cos^2\theta =$ L.H.S.

e L.H.S. $= \dfrac{1 - \tan^2 A}{\sec^2 A} \equiv \cos^2 A\left(1 - \dfrac{\sin^2 A}{\cos^2 A}\right)$
$\equiv \cos^2 A - \sin^2 A$
$\equiv (1 - \sin^2 A) - \sin^2 A$
$\equiv 1 - 2\sin^2 A =$ R.H.S.

f L.H.S. $= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta} \equiv \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$
$= \dfrac{1}{\cos^2\theta\sin^2\theta} \equiv \sec^2\theta\operatorname{cosec}^2\theta =$ R.H.S.

g L.H.S. $\equiv \operatorname{cosec} A(1 + \tan^2 A) \equiv \operatorname{cosec} A\left(1 + \dfrac{\sin^2 A}{\cos^2 A}\right)$
$\equiv \operatorname{cosec} A + \dfrac{1}{\sin A} \cdot \dfrac{\sin^2 A}{\cos^2 A}$
$\equiv \operatorname{cosec} A + \dfrac{\sin A}{\cos A} \cdot \dfrac{1}{\cos A}$
$\equiv \operatorname{cosec} A + \tan A \sec A =$ R.H.S.

h L.H.S. $= \sec^2\theta - \sin^2\theta \equiv (1 + \tan^2\theta) - (1 - \cos^2\theta)$
$= \tan^2\theta + \cos^2\theta =$ R.H.S.

7 $\dfrac{\sqrt{2}}{4}$

8 **a** 20.9°, 69.1°, 201°, 249°

b $\pm\frac{\pi}{3}$

c −153°, −135°, 26.6°, 45°

d $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

e 120°

f $0, \frac{\pi}{4}, \pi$

g 0°, 180°

h $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{4}, \frac{4\pi}{3}$

9 **a** $1+\sqrt{2}$

b $\cos k° = \dfrac{1}{1+\sqrt{2}} = \dfrac{\sqrt{2}-1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \sqrt{2}-1$

c 65.5°, 294.5°

10 **a** $b = \dfrac{4}{a}$

b $c^2 = \cot^2 x = \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{b^2}{1-b^2} = \dfrac{\left(\frac{4}{a}\right)^2}{1-\left(\frac{4}{a}\right)^2}$

$= \dfrac{16}{a^2} \times \dfrac{a^2}{(a^2-16)} = \dfrac{16}{a^2-16}$

11 **a** $\dfrac{1}{x} = \dfrac{1}{\sec\theta + \tan\theta} = \dfrac{\sec\theta - \tan\theta}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$

$= \dfrac{\sec\theta - \tan\theta}{(\sec^2\theta - \tan^2\theta)} = \dfrac{\sec\theta - \tan\theta}{1}$

b $x^2 + \dfrac{1}{x^2} + 2 = \left(x + \dfrac{1}{x}\right)^2 = (2\sec\theta)^2 = 4\sec^2\theta$

12 $p = 2(1+\tan^2\theta) - \tan^2\theta = 2 + \tan^2\theta$

$\Rightarrow \tan^2\theta = p - 2 \Rightarrow \cot^2\theta = \dfrac{1}{p-2}$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \dfrac{1}{p-2} = \dfrac{(p-2)+1}{p-2} = \dfrac{p-1}{p-2}$

Exercise C

1 Example: With $A = 60°$, $B = 30°$,

$\sin(A+B) = \sin 90° = 1$; $\sin A + \sin B = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \neq 1$

[You can find examples of A and B for which the statement is true, e.g. $A = 30°$, $B = -30°$, but one counter-example shows that it is not an identity.]

2 $\cos(\theta - \theta) = \cos\theta\cos\theta + \sin\theta\sin\theta$

$\Rightarrow \sin^2\theta + \cos^2\theta \equiv 1$ as $\cos\theta = 0$

3 **a** $\sin\left(\dfrac{\pi}{2} - \theta\right) = \sin\dfrac{\pi}{2}\cos\theta - \cos\dfrac{\pi}{2}\sin\theta$

$= (1)\cos\theta - (0)\sin\theta = \cos\theta$

b $\cos\left(\dfrac{\pi}{2} - \theta\right) = \cos\dfrac{\pi}{2}\cos\theta + \sin\dfrac{\pi}{2}\sin\theta$

$= (0)\cos\theta + (1)\sin\theta = \sin\theta$

4 a $\sin 35°$ **b** $\sin 35°$ **c** $\cos 210°$ **d** $\tan 31°$

e $\cos\theta$ **f** $\cos 7\theta$ **g** $\sin 3\theta$ **h** $\tan 5\theta$

i $\sin A$ **j** $\cos 3x$

5 a 1 **b** 0 **c** $\dfrac{\sqrt{3}}{2}$ **d** $\dfrac{\sqrt{2}}{2}$

e $\dfrac{\sqrt{2}}{2}$ **f** $-\frac{1}{2}$ **g** $\sqrt{3}$ **h** $\dfrac{\sqrt{3}}{3}$

i 1 **j** $\sqrt{2}$

6 a $(60-\theta)°$

b $\dfrac{\sin(60-\theta)°}{3} = \dfrac{\sin\theta°}{4}$

$\Rightarrow 4(\sin 60° \cos\theta° - \cos 60° \sin\theta°) = 3\sin\theta$

$\Rightarrow 4\left(\dfrac{\sqrt{3}}{2}\cos\theta° - \frac{1}{2}\sin\theta°\right) = 3\sin\theta$

$\Rightarrow 2\sqrt{3}\cos\theta° = 5\sin\theta° \Rightarrow \tan\theta° = \dfrac{2\sqrt{3}}{5}$

7 a $\text{L.H.S.} = \sin A\cos 60° + \cos A\sin 60°$
$\quad + \sin A\cos 60° - \cos A\sin 60°$
$= 2\sin A\cos 60°$
$= 2\sin A\left(\frac{1}{2}\right) = \sin A = \text{R.H.S.}$

b $\text{L.H.S.} = \dfrac{\cos A\cos B - \sin A\sin B}{\sin B\cos B} = \dfrac{\cos(A+B)}{\sin B\cos B}$
$= \text{R.H.S.}$

c $\text{L.H.S.} = \dfrac{\sin x\cos y + \cos x\sin y}{\cos x\cos y}$

$= \dfrac{\sin x\cancel{\cos y}}{\cos x\cancel{\cos y}} + \dfrac{\cancel{\cos x}\sin y}{\cancel{\cos x}\cos y}$

$= \tan x + \tan y = \text{R.H.S.}$

d $\text{L.H.S.} = \dfrac{\cos x\cos y - \sin x\sin y}{\sin x\sin y} + 1$

$= \cot x\cot y - 1 + 1 = \cot x\cot y = \text{R.H.S.}$

e $\text{L.H.S.} = \cos\theta\cos\dfrac{\pi}{3} - \sin\theta\sin\dfrac{\pi}{3} + \sqrt{3}\sin\theta$

$= \frac{1}{2}\cos\theta - \dfrac{\sqrt{3}}{2}\sin\theta + \sqrt{3}\sin\theta$

$= \frac{1}{2}\cos\theta + \dfrac{\sqrt{3}}{2}\sin\theta$

$= \sin\dfrac{\pi}{6}\cos\theta + \cos\dfrac{\pi}{6}\sin\theta$

$= \sin\left(\dfrac{\pi}{6} + \theta\right) = \text{R.H.S.}$

f $\text{R.H.S.} = \dfrac{\dfrac{1}{\tan A}\,\dfrac{1}{\tan B} - 1}{\dfrac{1}{\tan A} + \dfrac{1}{\tan B}} = \dfrac{1 - \tan A\tan B}{\tan A + \tan B}$

$= \dfrac{1}{\dfrac{\tan A + \tan B}{1 - \tan A\tan B}} = \dfrac{1}{\tan(A+B)}$

$= \cot(A+B)$

g L.H.S. $= [\sin 45^\circ \cos \theta^\circ + \cos 45^\circ \sin \theta^\circ]^2$
$+ [\sin 45^\circ \cos \theta^\circ - \cos 45^\circ \sin \theta^\circ]^2$
$= \frac{1}{2} [(\cos \theta^\circ + \sin \theta^\circ)^2 + (\cos \theta^\circ - \sin \theta^\circ)^2]$
$= \frac{1}{2} [\cos^2 \theta^\circ + \sin^2 \theta^\circ + 2 \sin \theta^\circ \cos \theta^\circ$
$+ \cos^2 \theta^\circ + \sin^2 \theta^\circ - 2 \sin \theta^\circ \cos \theta^\circ]$
$= \frac{1}{2} [1 + 1] = 1 =$ R.H.S.

h L.H.S. $= [\cos A \cos B - \sin A \sin B] \times$
$[\cos A \cos B + \sin A \sin B]$
$= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$
$= \cos^2 A \cos^2 B - (1 - \cos^2 A) \sin^2 B$
$= \cos^2 A (\cos^2 B + \sin^2 B) - \sin^2 B$
$= \cos^2 A - \sin^2 B =$ R.H.S.

8 **a** $\frac{3 + 4\sqrt{3}}{10}$ **b** $\frac{4 + 3\sqrt{3}}{10}$ **c** $\frac{10(3\sqrt{3} - 4)}{11}$

9 **a** $\frac{3}{5}$ **b** $\frac{4}{5}$
c $\frac{3 - 4\sqrt{3}}{10}$ **d** $\frac{1}{7}$

10 **a** $-\frac{77}{85}$ **b** $-\frac{36}{85}$ **c** $\frac{36}{77}$
11 **a** $-\frac{36}{325}$ **b** $\frac{204}{253}$ **c** $-\frac{325}{36}$
12 **a** $\cos 2\theta\ [= \cos(\theta + \theta)]$ **b** $\sin 8\theta\ [= \sin(4\theta + 4\theta)]$
c $\tan\left(\theta + \frac{\pi}{4}\right)$ **d** $\sin\left(\theta + \frac{\pi}{4}\right)$ or $\cos\left(\theta - \frac{\pi}{4}\right)$
13 **a** 51.7°, 231.7° **b** 170.1°, 350.1°
c 56.5°, 303.5° **d** 150°, 330°
e 153.4°, 161.6°, 333.4°, 341.6°
f 0°, 90°
14 **a** 30°, 270° **b** 30°, 270°

15 **a** $\tan(45 + 30)^\circ = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

b $\tan 75 = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{(3 + \sqrt{3})(3 + \sqrt{3})}{(3 - \sqrt{3})(3 + \sqrt{3})}$

$= \frac{12 + 6\sqrt{3}}{9 - 3} = 2 + \sqrt{3}$

16 $\cos 105^\circ = \cos(60 + 45)^\circ$
$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$
$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1 - \sqrt{3}}{2\sqrt{2}}$

so $\sec 105^\circ = \frac{2\sqrt{2}}{1 - \sqrt{3}} = \frac{2\sqrt{2}(1 + \sqrt{3})}{1 - 3} = -\sqrt{2}(1 + \sqrt{3})$

17 **a** $\frac{\sqrt{2}(\sqrt{3} + 1)}{4}$ **b** $\frac{\sqrt{2}(\sqrt{3} + 1)}{4}$

c $\frac{\sqrt{2}(\sqrt{3} - 1)}{4}$ **d** $\sqrt{3} - 2$

18 **a** $3(\sin x \cos y - \cos x \sin y)$
$- (\sin x \cos y + \cos x \sin y) = 0$
$\Rightarrow\ 2 \sin x \cos y - 4 \cos x \sin y = 0$
Divide throughout by $2 \cos x \cos y$
$\Rightarrow \tan x - 2 \tan y = 0$, so $\tan x = 2 \tan y$
b Using **a** $\tan x = 2 \tan y = 2 \tan 45^\circ = 2$
so $x = 63.4^\circ, 243.4^\circ$

19 2

20 $\dfrac{\tan x - 3}{3\tan x + 1}$

21 **a** $\frac{5}{3}$ **b** $\sqrt{3}$ **c** $-\left(\dfrac{8+5\sqrt{3}}{11}\right)$

22 **a** 45 **b** 225

23 $\cos y = \sin x \cos y + \sin y \cos x$
Divide by $\cos x \cos y \Rightarrow \sec x = \tan x + \tan y$,
so $\tan y = \sec x - \tan x$

24 $-\frac{6}{7}$

25 $\dfrac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} = \dfrac{1}{2} \Rightarrow (2+\sqrt{3})\tan x = 1 - 2\sqrt{3}$, so

$$\tan x = \frac{1-2\sqrt{3}}{2+\sqrt{3}} = \frac{(1-2\sqrt{3})(2-\sqrt{3})}{1} = 8 - 5\sqrt{3}$$

Exercise D

1 **a** $\sin 20°$ **b** $\cos 50°$ **c** $\cos 80°$
d $\tan 10°$ **e** $\operatorname{cosec} 49°$ **f** $3\cos 60°$
g $\frac{1}{2}\sin 16°$ **h** $\cos\left(\dfrac{\pi}{8}\right)$

2 **a** $\dfrac{\sqrt{2}}{2}$ **b** $\dfrac{\sqrt{3}}{2}$
c $\frac{1}{2}$ **d** 1

3 **a** $\cos 6\theta$ **b** $3\sin 4\theta$ **c** $\tan\theta$
d $2\cos\theta$ **e** $\sqrt{2}\cos\theta$ **f** $\frac{1}{4}\sin^2 2\theta$
g $\sin 4\theta$ **h** $-\frac{1}{2}\tan 2\theta$ **i** $\cos^2 2\theta$

4 $-\frac{7}{8}$

5 $\pm\frac{1}{5}$

6 mn

7 **a** **i** $\frac{24}{7}$ **ii** $\frac{24}{25}$ **iii** $\frac{7}{25}$
b $\frac{336}{625}$

8 **a** **i** $-\frac{7}{9}$ **ii** $\dfrac{2\sqrt{2}}{3}$ **iii** $-\dfrac{9\sqrt{2}}{8}$

b $\tan 2A = \dfrac{\sin 2A}{\cos 2A} = -\dfrac{4\sqrt{2}}{9} \times -\dfrac{9}{7} = \dfrac{4\sqrt{2}}{7}$

9 -3

10 51.3°

11 **a** $\cos 2\theta = \dfrac{3^2+6^2-5^2}{2\times3\times6} = \dfrac{20}{36} = \dfrac{5}{9}$ **b** $\dfrac{\sqrt{2}}{3}$

12 **a** $\dfrac{3}{4}$

b $m = \tan 2\theta = \dfrac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \dfrac{3}{2}\times\dfrac{16}{7} = \dfrac{24}{7}$

Exercise E

1 **a** $$\text{L.H.S.} = \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$$
$$= \frac{\cancel{(\cos A + \sin A)}(\cos A - \sin A)}{\cancel{\cos A + \sin A}}$$
$$= \cos A - \sin A = \text{R.H.S.}$$

b R.H.S. $= \dfrac{2}{2\sin A\cos A}\{\sin B\cos A - \cos B\ \sin A\}$

$= \dfrac{\sin B}{\sin A} - \dfrac{\cos B}{\cos A} = \text{L.H.S.}$

c L.H.S. $= \dfrac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} = \dfrac{\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2}\cancel{\sin\theta}\cos\theta}$

$= \tan\theta = \text{R.H.S.}$

d L.H.S. $= \dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1+\dfrac{\sin^2\theta}{\cos^2\theta}}{1-\dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta} = \dfrac{1}{\cos 2\theta} = \sec 2\theta = \text{R.H.S.}$

e L.H.S. $= 2\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)$

$= 2\sin\theta\cos\theta = \sin 2\theta = \text{R.H.S.}$

f L.H.S. $= \dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin(3\theta-\theta)}{\sin\theta\cos\theta} = \dfrac{\sin 2\theta}{\sin\theta\cos\theta}$

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{R.H.S.}$

g L.H.S. $= \dfrac{1}{\sin\theta} - \dfrac{2\cos 2\theta\cos\theta}{\sin 2\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{2\cos 2\theta\cancel{\cos\theta}}{2\sin\theta\cancel{\cos\theta}} = \dfrac{1-\cos 2\theta}{\sin\theta}$

$= \dfrac{1-(1-2\sin^2\theta)}{\sin\theta} = 2\sin\theta = \text{R.H.S.}$

h L.H.S. $= \dfrac{\dfrac{1}{\cos\theta}-1}{\dfrac{1}{\cos\theta}+1} = \dfrac{1-\cos\theta}{1+\cos\theta}$

$= \dfrac{1-\left(1-2\sin^2\dfrac{\theta}{2}\right)}{1+\left(2\cos^2\dfrac{\theta}{2}-1\right)} = \dfrac{\cancel{2}\sin^2\dfrac{\theta}{2}}{\cancel{2}\cos^2\dfrac{\theta}{2}} = \tan^2\dfrac{\theta}{2}$

$= \text{R.H.S.}$

i L.H.S. $= \dfrac{1-\tan x}{1+\tan x} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$

$= \dfrac{(\cos x - \sin x)(\cos x - \sin x)}{\cos^2 x - \sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x - 2\sin x\cos x}{\cos^2 x - \sin^2 x}$

$= \dfrac{1-\sin 2x}{\cos 2x} = \text{R.H.S.}$

2 a L.H.S. $= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$$= \frac{1}{(\frac{1}{2})\sin 2\theta} = 2\operatorname{cosec} 2\theta = \text{R.H.S.}$$

b 4

3 a $0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}, 2\pi$

b $\pm 38.7°$

c 30, 150, 210, 330

d $\dfrac{\pi}{12}, \dfrac{\pi}{4}, \dfrac{5\pi}{12}, \dfrac{3\pi}{4}$

e 33.2°, 146.8°, 213.2°, 326.8°

f 60°, 300°, 443.6°, 636.4°

g $\dfrac{\pi}{8}, \dfrac{5\pi}{8}$

h $\dfrac{\pi}{4}, \dfrac{5\pi}{4}$

i 0°, 30°, 150°, 180°, 210°, 330°

j $\dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{6}, \dfrac{5\pi}{3}$

k 0°, ±113.6°, 180°

l −104.0°, 0°, 76.0°, 180°

m 0°, 35.3°, 144.7°, 180°, 215.3°, 324.7°

4 $q = \dfrac{p^2}{2} - 1$

5 a $y = 2(1 - x)$

b $2xy = 1 - x^2$

c $y^2 = 4x^2(1 - x^2)$

d $y^2 = \dfrac{2(4 - x)}{3}$

6 a L.H.S. $= \cos^2 2\theta + \sin^2 2\theta - 2\sin 2\theta\cos 2\theta$

$= 1 - \sin 4\theta = \text{R.H.S.}$

b $\dfrac{\pi}{24}, \dfrac{17\pi}{24}$

7 a i R.H.S. $= \dfrac{2\tan\left(\frac{\theta}{2}\right)}{\sec^2\left(\frac{\theta}{2}\right)} = 2\dfrac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} \times \dfrac{\cos^2\left(\frac{\theta}{2}\right)}{1}$

$$= 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \sin\theta$$

ii R.H.S. $= \dfrac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)} = \dfrac{1 - \tan^2\left(\frac{\theta}{2}\right)}{\sec^2\left(\frac{\theta}{2}\right)}$

$$= \cos^2\left(\frac{\theta}{2}\right)\left\{1 - \tan^2\left(\frac{\theta}{2}\right)\right\}$$

$$= \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = \cos\theta = \text{L.H.S.}$$

b **i** 90°, 323.1°

ii 13.3°, 240.4°

8 **a** **i** $\cos x = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow 2\cos^2\frac{x}{2} \equiv 1 + \cos x \Rightarrow \cos^2\frac{x}{2} \equiv \frac{1+\cos x}{2}$

ii $\cos x = 1 - 2\sin^2\frac{x}{2}$

$\Rightarrow 2\sin^2\frac{x}{2} \equiv 1 - \cos x \Rightarrow \sin^2\frac{x}{2} \equiv \frac{1-\cos x}{2}$

b **i** $\frac{2\sqrt{5}}{5}$ **ii** $\frac{\sqrt{5}}{5}$ **iii** $\frac{1}{2}$

c $\cos^4\frac{A}{2} = \left(\frac{1+\cos A}{2}\right)^2 = \frac{1 + 2\cos A + \cos^2 A}{4}$

$$= \frac{1 + 2\cos A + \left(\frac{1+\cos 2A}{2}\right)}{4}$$

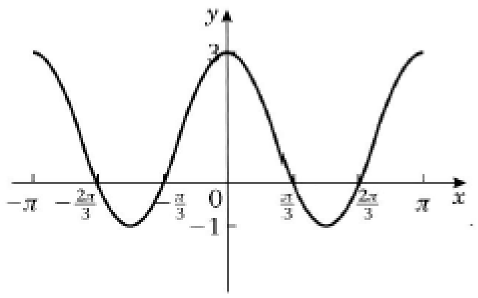
$$= \frac{2 + 4\cos A + 1 + \cos 2A}{8}$$

$$= \frac{3 + 4\cos A + \cos 2A}{8}$$

9 **a** L.H.S. $= \frac{3(1+\cos 2x)}{2} - \frac{(1-\cos 2x)}{2}$

$= 1 + 2\cos 2x$

b



Crosses y-axis at (0, 3)

Crosses x-axis at

$\left(-\frac{2\pi}{3}, 0\right), \left(-\frac{\pi}{3}, 0\right), \left(\frac{\pi}{3}, 0\right), \left(\frac{2\pi}{3}, 0\right)$

10 **a** $2\cos^2\left(\frac{\theta}{2}\right) - 4\sin^2\left(\frac{\theta}{2}\right) = 2\left(\frac{1+\cos\theta}{2}\right) - 4\left(\frac{1-\cos\theta}{2}\right)$

$= 1 + \cos\theta - 2 + 2\cos\theta$

$= 3\cos\theta - 1$

b 131.8°, 228.2°

11 **a** $(\sin^2 A + \cos^2 A)^2 = \sin^4 A + \cos^4 A + 2\sin^2 A\cos^2 A$

So $1 = \sin^4 A + \cos^4 A + \frac{(2\sin A\cos A)^2}{2}$

$\Rightarrow 2 = 2(\sin^4 A + \cos^4 A) + \sin^2 2A$

$\sin^4 A + \cos^4 A = \frac{1}{2}(2 - \sin^2 2A)$

b Using **a**: $\sin^4 A + \cos^4 A = \frac{1}{2}(2 - \sin^2 2A)$

$$= \frac{1}{2}\left\{2 - \frac{(1 - \cos 4A)}{2}\right\}$$

$$= \frac{(4 - 1 + \cos 4A)}{4}$$

$$= \frac{3 + \cos 4A}{4}$$

c $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

12 a $\cos(2A + A) = \cos 2A \cos A - \sin 2A \sin A$

$$= (\cos^2 A - \sin^2 A)\cos A - 2\sin^2 A \cos A$$

$$= \cos^3 A - 3\sin^2 A \cos A$$

$$= \cos^3 A - 3(1 - \cos^2 A)\cos A$$

$$= \cos^3 A - 3\cos A + 3\cos^3 A$$

$$= 4\cos^3 A - 3\cos A$$

b 20°, 100°, 140°, 220°, 260°, 340°

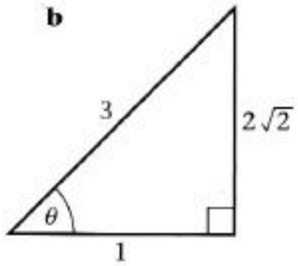
13 a $\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta} = \frac{3\sin\theta\cos^2\theta - \sin^3\theta}{\cos^3\theta - 3\sin^2\theta\cos\theta}$

Use the intermediate step result, that includes $\sin\theta$ and $\cos\theta$, for $\sin 3\theta$ and $\cos 3\theta$. See Example 13 for $\sin 3\theta$, and the solution to question 12 for $\cos 3\theta$.

$$= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Divide 'top and bottom' by $\cos^3\theta$.

b



$\tan\theta = 2\sqrt{2}$

$$\text{so } \tan 3\theta = \frac{6\sqrt{2} - 16\sqrt{2}}{1 - 24} = \frac{-10\sqrt{2}}{-23} = \frac{10\sqrt{2}}{23}$$

Exercise F

1 $R = 13$; $\tan\alpha = \frac{12}{5}$
2 35.3°
3 41.8°
4 a R.H.S. $= \sqrt{2}\left\{\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right\}$

$$= \sqrt{2}\left\{\sin\theta\,\frac{1}{\sqrt{2}} + \cos\theta\,\frac{1}{\sqrt{2}}\right\}$$

$$= \sin\theta + \cos\theta = \text{L.H.S.}$$

b R.H.S. $= 2\left\{\sin 2\theta \cos \frac{\pi}{6} - \cos 2\theta \sin \frac{\pi}{6}\right\}$

$= 2\left\{\sin 2\theta \, \frac{\sqrt{3}}{2} - \cos 2\theta \, \frac{1}{2}\right\}$

$= \sqrt{3} \sin 2\theta - \cos 2\theta =$ L.H.S.

5 $2\cos\left(2\theta + \frac{\pi}{3}\right) = 2\left(\cos 2\theta \cos \frac{\pi}{3} - \sin 2\theta \sin \frac{\pi}{3}\right)$

$= 2\left(\cos 2\theta \, \frac{1}{2} - \sin 2\theta \frac{\sqrt{3}}{2}\right)$

$\equiv \cos 2\theta - \sqrt{3} \sin 2\theta$

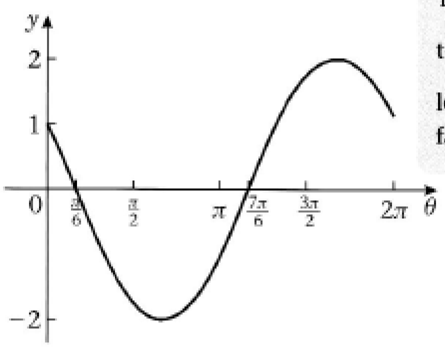
The other part follows from **4b**

6 a $R = \sqrt{10}$, $\alpha = 71.6°$ **b** $R = 5$, $\alpha = 53.1°$

c $R = \sqrt{53}$, $\alpha = 74.1°$ **d** $R = \sqrt{5}$, $\alpha = 63.4°$

7 a $\cos\theta - \sqrt{3}\sin\theta \equiv R\cos(\theta + \alpha)$ gives $R = 2$, $\alpha = \frac{\pi}{3}$

b $y = 2\cos\left(\theta + \frac{\pi}{3}\right)$



The graph of $y = \cos\theta$ translated by $\frac{\pi}{3}$ to the left and stretched by factor of 2 vertically.

8 a $3\sin 3\theta - 4\cos 3\theta \equiv R\sin(3\theta - \alpha)$

$\equiv R\sin 3\theta \cos\alpha - R\cos 3\theta \sin\alpha$

So $R\cos\alpha = 3$, $R\sin\alpha = 4 \Rightarrow R = 5$,

$\tan\alpha = \frac{4}{3}$ so $\alpha = \tan^{-1}\frac{4}{3} = 53.1°$

b Minimum value is -5,

when $3\theta - 53.1 = 270 \Rightarrow \theta = 107.7$

9 a $\sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right)$ **b** $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

10 a $25\cos(\theta + 73.7°)$ **b** $(0, 7)$

c $25, -25$ **d i** 2 **ii** 0 **iii** 1

11 a $5\left(\frac{1 - \cos 2\theta}{2}\right) - 3\left(\frac{1 + \cos 2\theta}{2}\right) + 3\sin 2\theta$

$\equiv 1 + 3\sin 2\theta - 4\cos 2\theta$, so $a = 3$, $b = -4$, $c = 1$

b Maximum = 6, minimum = −4

12 a 6.9°, 66.9° **b** 16.6°, 65.9°

c 8.0°, 115.9° **d** −165.2°, 74.8°

13 a 39.1°, 152.2°, 219.1°, 332.2°

b 126.9° **c** 31.7°, 121.7° **d** $\frac{\pi}{3}$

14 a 125.3°, 305.3° **b** 109.5° **c** 70.5°

No solutions for **d** and **e**

15 a $R = \sqrt{10}$, $\alpha = 18.4°$, $\theta = 69.2°, 327.7°$

b $9\cos^2\theta = 4 - 4\sin\theta + \sin^2\theta$

$\Rightarrow 9(1 - \sin^2\theta) = 4 - 4\sin\theta + \sin^2\theta$

So $10\sin^2\theta - 4\sin\theta - 5 = 0$

c 69.2°, 110.8°, 212.3°, 327.7°

d When you square you are also solving $3\cos\theta = -(2 - \sin\theta)$. The other two solutions are for this equation.