Trigonometry

Exercise A

1 Solve the following equations for θ, in the interval $0 < \theta \leqslant 360°$:

a $\sin\theta = -1$

b $\tan\theta = \sqrt{3}$

c $\cos\theta = \frac{1}{2}$

d $\sin\theta = \sin 15°$

e $\cos\theta = -\cos 40°$

f $\tan\theta = -1$

g $\cos\theta = 0$

h $\sin\theta = -0.766$

i $7\sin\theta = 5$

j $2\cos\theta = -\sqrt{2}$

k $\sqrt{3}\sin\theta = \cos\theta$

l $\sin\theta + \cos\theta = 0$

m $3\cos\theta = -2$

n $(\sin\theta - 1)(5\cos\theta + 3) = 0$

o $\tan\theta = \tan\theta(2 + 3\sin\theta)$

2 Solve the following equations for x, giving your answers to 3 significant figures where appropriate, in the intervals indicated:

a $\sin x° = -\frac{\sqrt{3}}{2}$, $-180 \leqslant x \leqslant 540$

b $2\sin x° = -0.3$, $-180 \leqslant x \leqslant 180$

c $\cos x° = -0.809$, $-180 \leqslant x \leqslant 180$

d $\cos x° = 0.84$, $-360 < x < 0$

e $\tan x° = -\frac{\sqrt{3}}{3}$, $0 \leqslant x \leqslant 720$

f $\tan x° = 2.90$, $80 \leqslant x \leqslant 440$

3.

Find the values of θ, in the interval $0 \leqslant \theta \leqslant 360°$, for which:

a $\sin 4\theta = 0$ **b** $\cos 3\theta = -1$ **c** $\tan 2\theta = 1$ **d** $\cos 2\theta = \frac{1}{2}$

e $\tan\frac{1}{2}\theta = -\frac{1}{\sqrt{3}}$ **f** $\sin(-\theta) = \frac{1}{\sqrt{2}}$ **g** $\tan(45° - \theta) = -1$

h $2\sin(\theta - 20°) = 1$ **i** $\tan(\theta + 75°) = \sqrt{3}$ **j** $\cos(50° + 2\theta) = -1$

Exercise B

1 Simplify each of the following expressions:

a $1 - \cos^2 \frac{1}{2}\theta$ **b** $5\sin^2 3\theta + 5\cos^2 3\theta$ **c** $\sin^2 A - 1$

d $\dfrac{\sin\theta}{\tan\theta}$ **e** $\dfrac{\sqrt{1-\cos^2 x}}{\cos x}$ **f** $\dfrac{\sqrt{1-\cos^2 3A}}{\sqrt{1-\sin^2 3A}}$

g $(1+\sin x)^2 + (1-\sin x)^2 + 2\cos^2 x$

h $\sin^4\theta + \sin^2\theta\cos^2\theta$

i $\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta$

2 Given that $2\sin\theta = 3\cos\theta$, find the value of $\tan\theta$.

3 Given that $\sin x\cos y = 3\cos x\sin y$, express $\tan x$ in terms of $\tan y$.

4 Express in terms of $\sin\theta$ only:

a $\cos^2\theta$ **b** $\tan^2\theta$ **c** $\cos\theta\tan\theta$

d $\dfrac{\cos\theta}{\tan\theta}$ **e** $(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

5 Using the identities $\sin^2 A + \cos^2 A \equiv 1$ and/or $\tan A \equiv \dfrac{\sin A}{\cos A}$ $(\cos A \neq 0)$, prove that:

a $(\sin\theta + \cos\theta)^2 \equiv 1 + 2\sin\theta\cos\theta$

b $\dfrac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

c $\tan x + \dfrac{1}{\tan x} \equiv \dfrac{1}{\sin x\cos x}$

d $\cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$

e $(2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2 \equiv 5$

f $2 - (\sin\theta - \cos\theta)^2 \equiv (\sin\theta + \cos\theta)^2$

g $\sin^2 x\cos^2 y - \cos^2 x\sin^2 y = \sin^2 x - \sin^2 y$

6 Find, without using your calculator, the values of:

a $\sin\theta$ and $\cos\theta$, given that $\tan\theta = \frac{5}{12}$ and θ is acute.

b $\sin\theta$ and $\tan\theta$, given that $\cos\theta = -\frac{3}{5}$ and θ is obtuse.

c $\cos\theta$ and $\tan\theta$, given that $\sin\theta = -\frac{7}{25}$ and $270° < \theta < 360°$.

Exercise C

Solve for θ, in the interval $0 \leqslant \theta \leqslant 360°$, the following equations.
Give your answers to 3 significant figures where they are not exact.

a $4\cos^2\theta = 1$ **b** $2\sin^2\theta - 1 = 0$

c $3\sin^2\theta + \sin\theta = 0$ **d** $\tan^2\theta - 2\tan\theta - 10 = 0$

e $2\cos^2\theta - 5\cos\theta + 2 = 0$ **f** $\sin^2\theta - 2\sin\theta - 1 = 0$

g $\tan^2 2\theta = 3$ **h** $4\sin\theta = \tan\theta$

i $\sin\theta + 2\cos^2\theta + 1 = 0$ **j** $\tan^2(\theta - 45°) = 1$

k $3\sin^2\theta = \sin\theta\cos\theta$ **l** $4\cos\theta(\cos\theta - 1) = -5\cos\theta$

m $4(\sin^2\theta - \cos\theta) = 3 - 2\cos\theta$ **n** $2\sin^2\theta = 3(1 - \cos\theta)$

o $4\cos^2\theta - 5\sin\theta - 5 = 0$ **p** $\cos^2\dfrac{\theta}{2} = 1 + \sin\dfrac{\theta}{2}$