Polynomials and Factor Theorem

Exercise A

1 Divide:

   a $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$ **b** $x^3 + 10x^2 + 25x + 4$ by $(x + 4)$

   c $x^3 + 7x^2 - 3x - 54$ by $(x + 6)$ **d** $x^3 + 9x^2 + 18x - 10$ by $(x + 5)$

   e $x^3 - x^2 + x + 14$ by $(x + 2)$ **f** $x^3 + x^2 - 7x - 15$ by $(x - 3)$

   g $x^3 - 5x^2 + 8x - 4$ by $(x - 2)$ **h** $x^3 - 3x^2 + 8x - 6$ by $(x - 1)$

   i $x^3 - 8x^2 + 13x + 10$ by $(x - 5)$ **j** $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$

2 Divide:

   a $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$ **b** $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

   c $3x^3 - 10x^2 - 10x + 8$ by $(x - 4)$ **d** $3x^3 - 5x^2 - 4x - 24$ by $(x - 3)$

   e $2x^3 + 4x^2 - 9x - 9$ by $(x + 3)$ **f** $2x^3 - 15x^2 + 14x + 24$ by $(x - 6)$

   g $-3x^3 + 2x^2 - 2x - 7$ by $(x + 1)$ **h** $-2x^3 + 5x^2 + 17x - 20$ by $(x - 4)$

   i $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$ **j** $-4x^3 + 9x^2 - 3x + 2$ by $(x - 2)$

3 Divide:

   a $x^4 + 5x^3 + 2x^2 - 7x + 2$ by $(x + 2)$

   b $x^4 + 11x^3 + 25x^2 - 29x - 20$ by $(x + 5)$

   c $4x^4 + 14x^3 + 3x^2 - 14x - 15$ by $(x + 3)$

   d $3x^4 - 7x^3 - 23x^2 + 14x - 8$ by $(x - 4)$

   e $-3x^4 + 9x^3 - 10x^2 + x + 14$ by $(x - 2)$

   f $3x^5 + 17x^4 + 2x^3 - 38x^2 + 5x - 25$ by $(x + 5)$

   g $6x^5 - 19x^4 + x^3 + x^2 + 13x + 6$ by $(x - 3)$

   h $-5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7$ by $(x - 1)$

   i $2x^6 - 11x^5 + 14x^4 - 16x^3 + 36x^2 - 10x - 24$ by $(x - 4)$

   j $-x^6 + 4x^5 - 4x^4 + 4x^3 - 5x^2 + 7x - 3$ by $(x - 3)$

Exercise B

1 Divide:

   a $x^3 + x + 10$ by $(x + 2)$

   b $2x^3 - 17x + 3$ by $(x + 3)$

   c $-3x^3 + 50x - 8$ by $(x - 4)$

2 Divide:

   a $x^3 + x^2 - 36$ by $(x - 3)$

   b $2x^3 + 9x^2 + 25$ by $(x + 5)$

   c $-3x^3 + 11x^2 - 20$ by $(x - 2)$

> **Hint for question 2:**
> Use $0x$.

3 Divide:

   a $x^3 + 2x^2 - 5x - 10$ by $(x + 2)$

   b $2x^3 - 6x^2 + 7x - 21$ by $(x - 3)$

   c $-3x^3 + 21x^2 - 4x + 28$ by $(x - 7)$

4 Find the remainder when:

    a $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

    b $3x^3 - 20x^2 + 10x + 5$ is divided by $(x - 6)$

    c $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

5 Show that when $3x^3 - 2x^2 + 4$ is divided by $(x - 1)$ the remainder is 5.

6 Show that when $3x^4 - 8x^3 + 10x^2 - 3x - 25$ is divided by $(x + 1)$ the remainder is -1.

7 Show that $(x + 4)$ is a factor of $5x^3 - 73x + 28$.

8 Simplify $\dfrac{3x^3 - 8x - 8}{x - 2}$.

> **Hint for question 8:**
> Divide $3x^3 - 8x - 8$ by $(x - 2)$.

9 Divide $x^3 - 1$ by $(x - 1)$.

> **Hint for question 9:**
> Use $0x^2$ and $0x$.

10 Divide $x^4 - 16$ by $(x + 2)$.

Exercise C

1 Use the factor theorem to show:

    a $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

    b $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

    c $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$

2 Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely.

3 Show that $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$ and hence factorise the expression completely.

4 Show that $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ and hence factorise the expression completely.

5 Show that $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$ and hence factorise the expression completely.

6 Each of these expressions has a factor $(x \pm p)$. Find a value of p and hence factorise the expression completely.

    a $x^3 - 10x^2 + 19x + 30$ **b** $x^3 + x^2 - 4x - 4$ **c** $x^3 - 4x^2 - 11x + 30$

7 Factorise:

    a $2x^3 + 5x^2 - 4x - 3$ **b** $2x^3 - 17x^2 + 38x - 15$

    c $3x^3 + 8x^2 + 3x - 2$ **d** $6x^3 + 11x^2 - 3x - 2$

    e $4x^3 - 12x^2 - 7x + 30$

8 Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$ find the value of a.

9 Given that $(x + 3)$ is a factor of $6x^3 - bx^2 + 18$ find the value of b.

10 Given that $(x - 1)$ and $(x + 1)$ are factors of $px^3 + qx^2 - 3x - 7$ find the value of p and q.

> **Hint for question 10:**
> Solve simultaneous equations.