

Cubic Graphs and Reciprocal Graphs ①

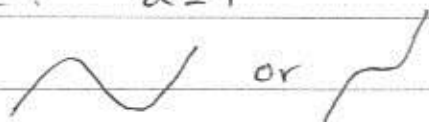
Answers

Exercise A

①

(a) $y = (x-3)(x-2)(x+1)$

shape: $a=1$



x-intercepts: sub $y=0$

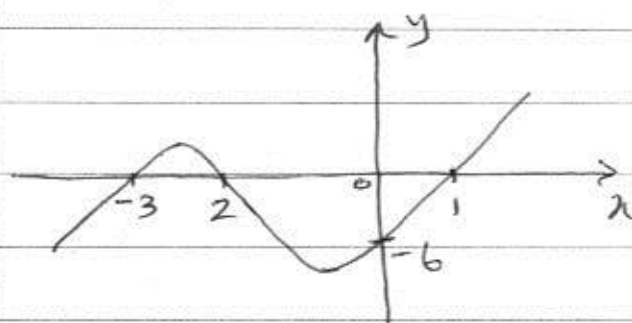
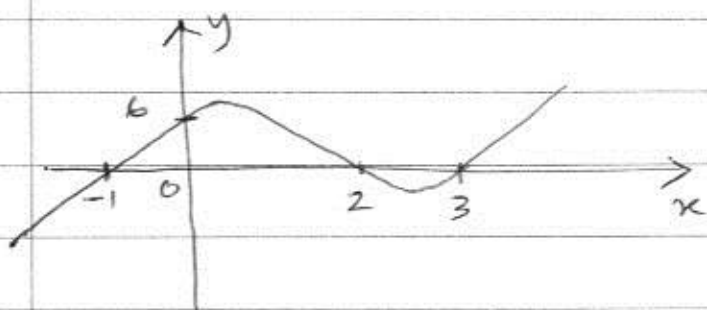
$$(x-3)(x-2)(x+1) = 0$$

$$x = 3, 2, -1$$

y-intercept: sub $x=0$

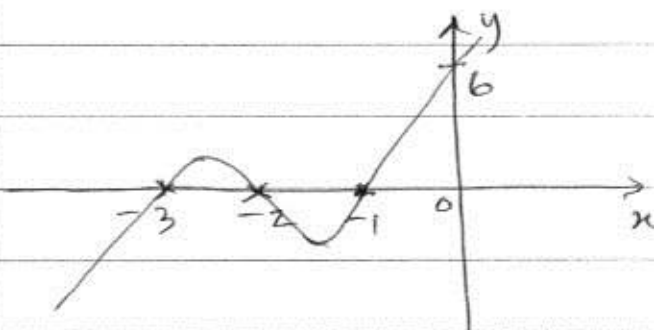
$$y = (-3)(-2)(1)$$

$$= 6$$



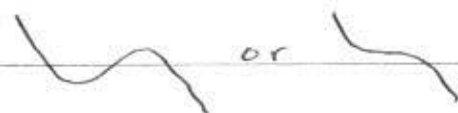
(c) $y = (x+1)(x+2)(x+3)$

This is similar to
①(a) and (b).



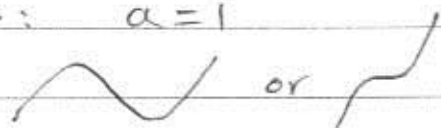
(d) $y = (x+1)(1-x)(x+3)$

shape: $a=-1$



(b) $y = (x-1)(x+2)(x+3)$

Shape: $a=1$



x-intercepts:

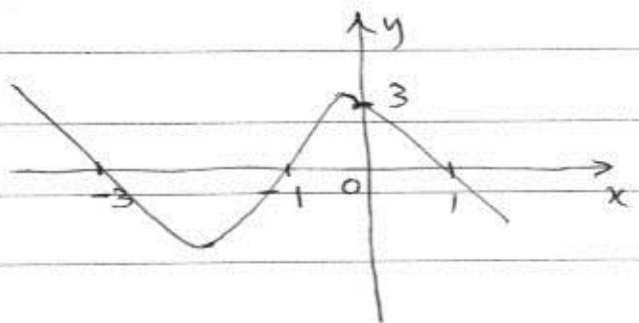
$$x = 1, x = -2, x = -3$$

y-intercept: $y = (-1)(2)(3)$
 $= -6$

x-intercepts:

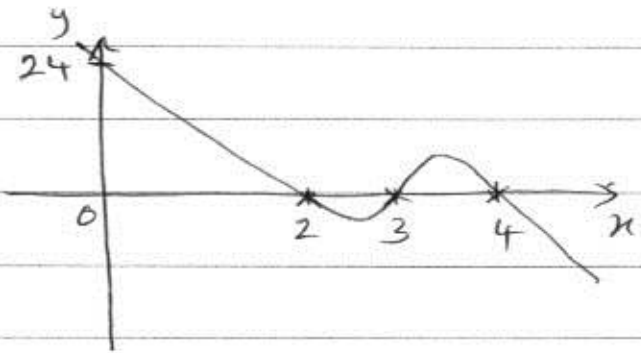
$$x = -1, 1, -3$$

y-intercept = $(1)(1)(3) = 3$



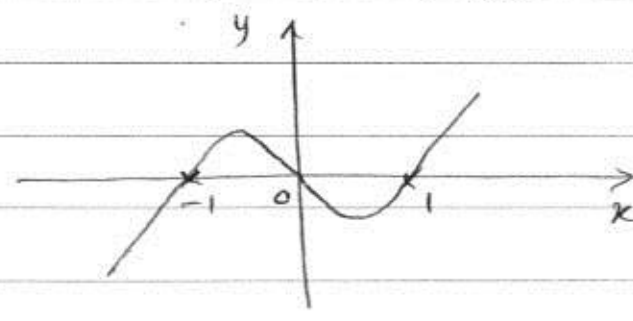
(e) $y = (x-2)(x-3)(4-x)$

This is similar to Q1(d).



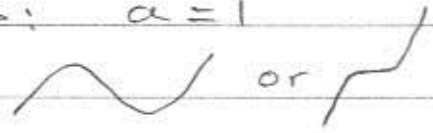
(g) $y = x(x+1)(x-1)$

Similar to Q1(f)



(f) $y = x(x-2)(x+1)$

Shape: $a = 1$



x-intercepts: sub $y = 0$

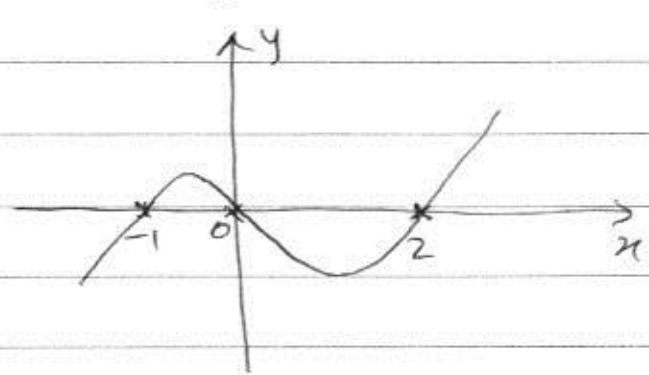
$x(x-2)(x+1) = 0$

$x = 0$ or $x-2 = 0$ or $x+1 = 0$

$x = 0, 2, -1$

y-intercept: sub $x = 0$

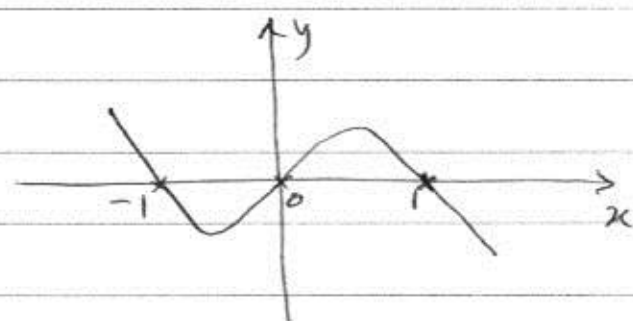
$y = 0$



(h) $y = x(x+1)(1-x)$

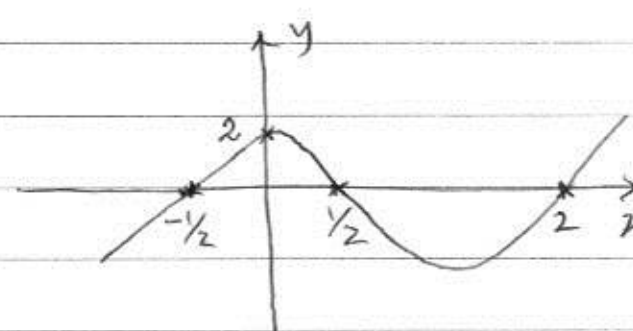
(Negative shape)

$a = -1$



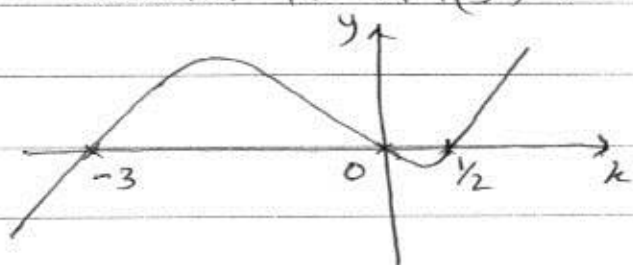
(i) $y = (x-2)(2x-1)(2x+1)$

Similar to Q1(a)



(j) $y = x(2x-1)(x+3)$

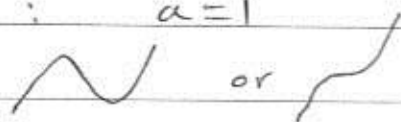
Similar to Q1(f)



2

a

(a) $y = (x+1)^2(x-1)$

Shape: $a=1$


x-intercepts:

$$(x+1)^2(x-1) = 0$$

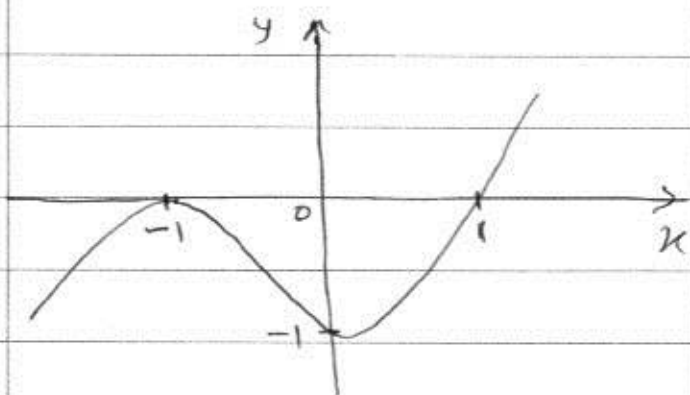
$$x = -1, \quad x = 1$$

↓
 Repeated root
 ↓
 Touches x-axis

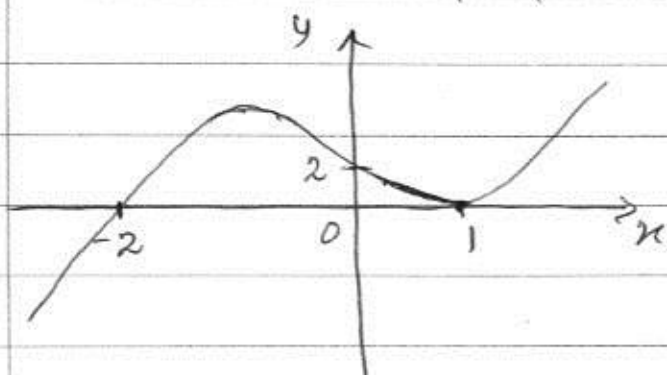
↓
 Crosses x-axis.

y-intercept:

$$y = (1)^2(-1) = -1$$

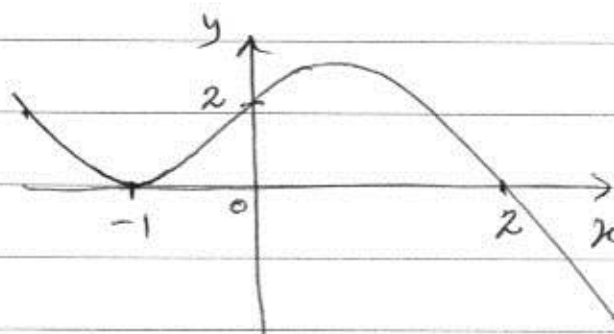


(b) $y = (x+2)(x-1)^2$
 Similar to Q2(a)

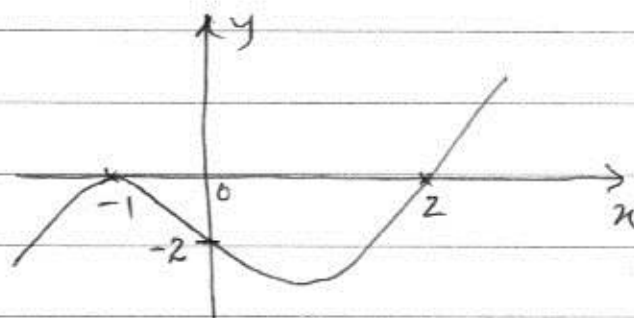


(c) $y = (2-x)(x+1)^2$

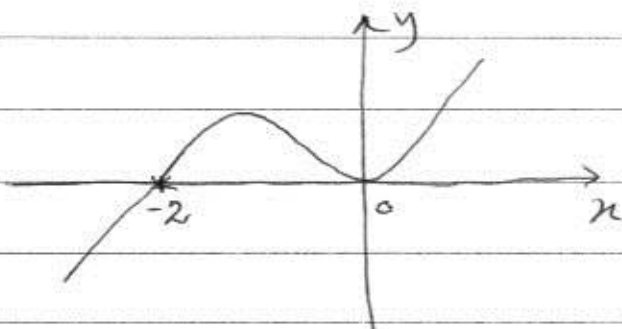
Negative shape, otherwise similar to Q2(a).



(d) $y = (x-2)(x+1)^2$
 Similar to Q2(a)

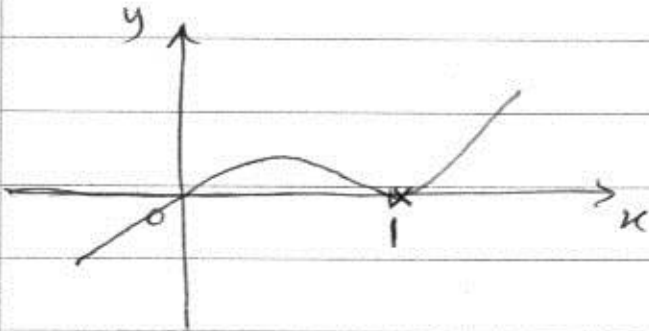


(e) $y = x^2(x+2)$
 Similar to Q2(a)

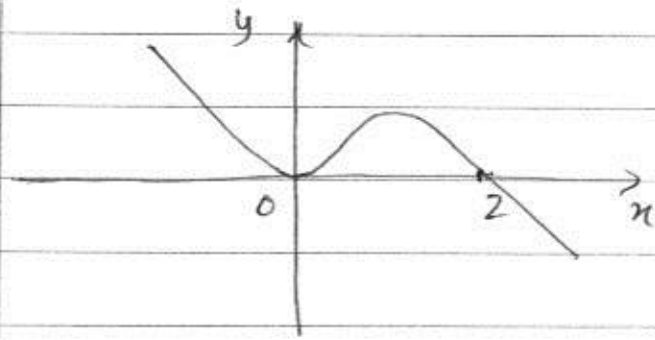


(f) $y = (x-1)^2 x$

Similar to Q2(a)

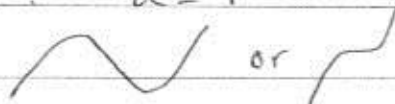


(i) $y = x^2(2-x)$

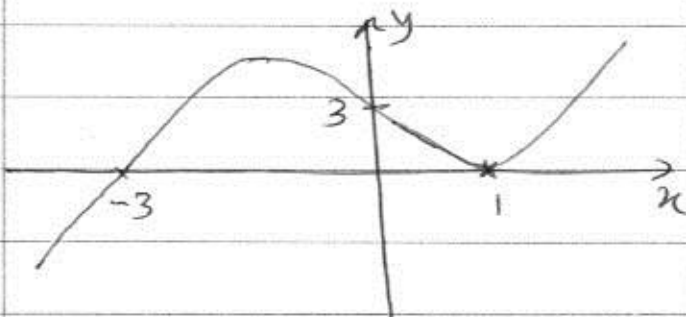


(g) $y = (1-x)^2(3+x)$

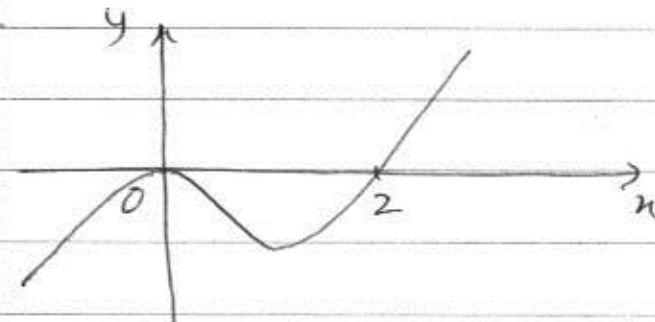
Shape: $a=1$



∴ Similar to Q2(a)



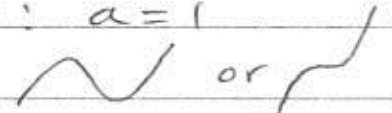
(j) $y = x^2(x-2)$



3

(a) $y = x^3 + x^2 - 2x$

Shape: $a=1$



x-intercepts: sub $y=0$

$$x^3 + x^2 - 2x = 0$$

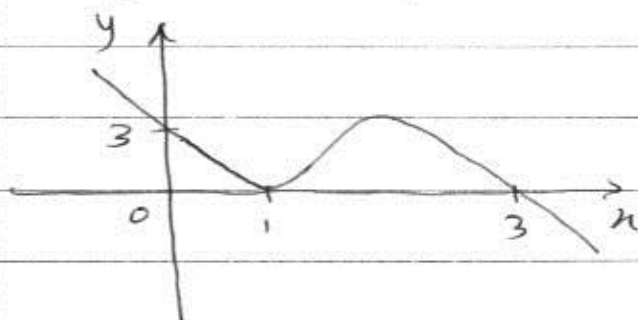
$$x(x^2 + x - 2) = 0$$

$$x(x+2)(x-1) = 0$$

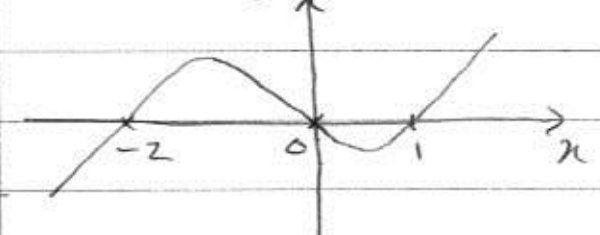
$$x = 0, 1, -2$$

y-intercept: sub $x=0$

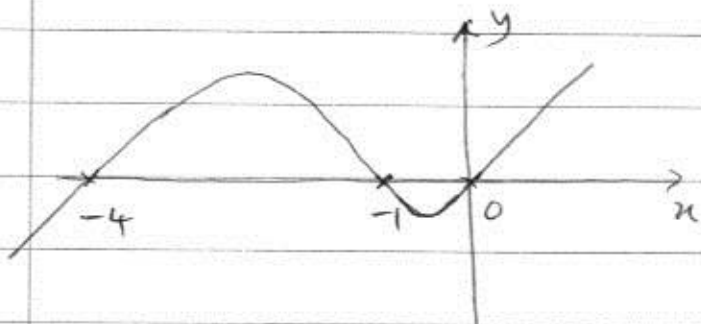
(h) $y = (x-1)^2(3-x)$
 Similar to Q2(a) but
 negative shape.



$y = 0$



(b) $y = x^3 + 5x^2 + 4x$
 Similar to Q3(a)

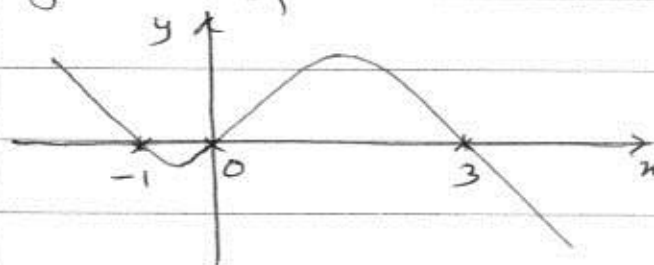


$$x(x^2 - 2x - 3) = 0$$

$$x(x-3)(x+1) = 0$$

$$x = 0, -1, 3$$

$$y\text{-intercept} = 0$$



(c) $y = x^3 + 2x^2 + x$

x -intercepts:

$$x^3 + 2x^2 + x = 0$$

$$x(x^2 + 2x + 1) = 0$$

$$x(x+1)(x+1) = 0$$

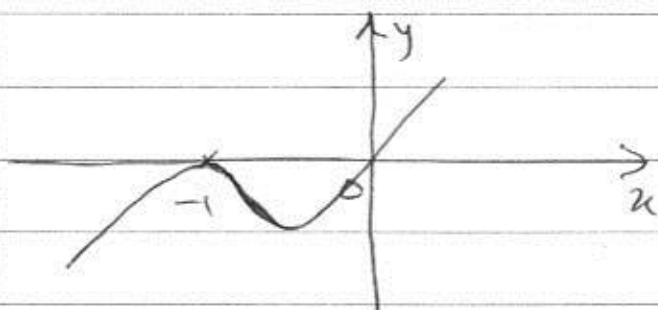
$$x = 0 \text{ or } x = -1$$



Crosses



Touches



(e) $y = x^3 - x^2$

x -intercept:

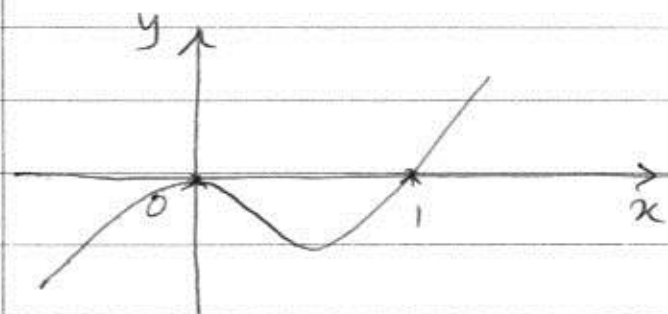
$$x^3 - x^2 = 0$$

$$x^2(x-1) = 0$$

$$x = 0, x = 1$$

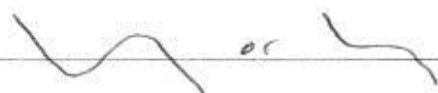
↓
Touches

↓
Crosses



(d) $y = 3x + 2x^2 - x^3$

Shape: $a = -1$

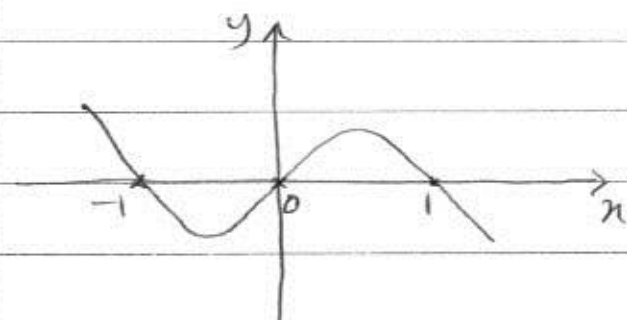


x -intercepts:

$$3x + 2x^2 - x^3 = 0$$

$$x^3 - 2x^2 - 3x = 0$$

(f) $y = x - x^3$



* These questions can be done
 * by transforming $y=x^3$. That's what Q1 tries to show.
 We will study that later.



Exercise B

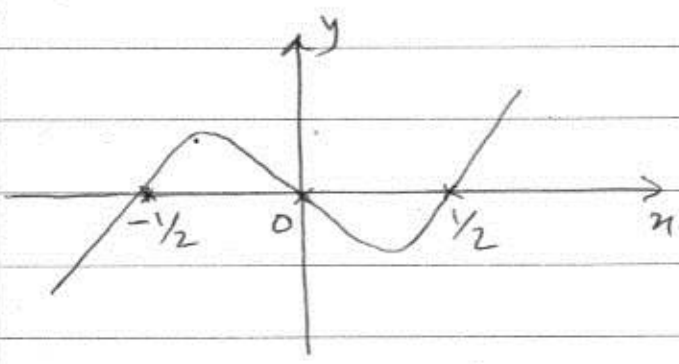
①

(a) $y = (x-2)^3$

shape: $a=1$

tries to show.
 We will study that later.

(g) $y = 12x^3 - 3x$



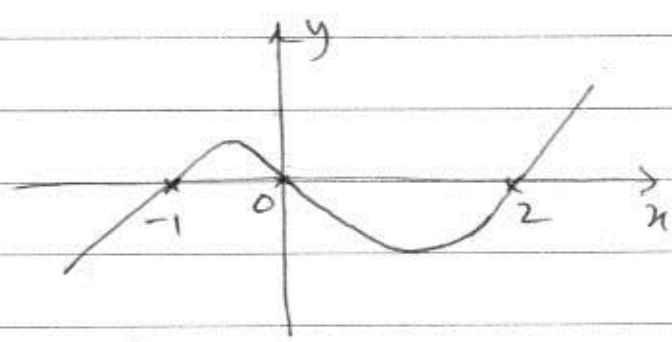
x-intercept: sub $y=0$

$$(x-2)^3 = 0$$

$$x-2=0$$

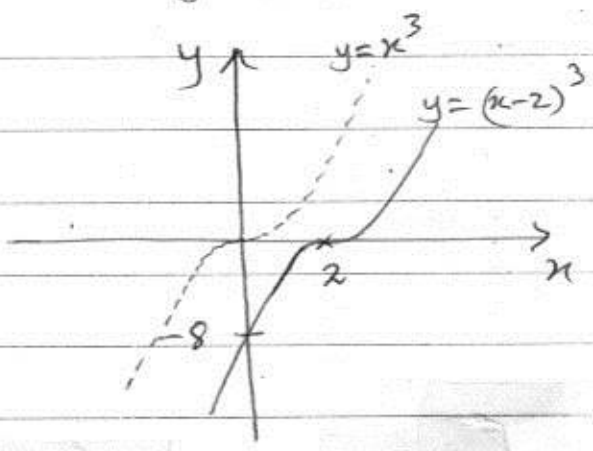
$$x=2$$

(h) $y = x^3 - x^2 - 2x$

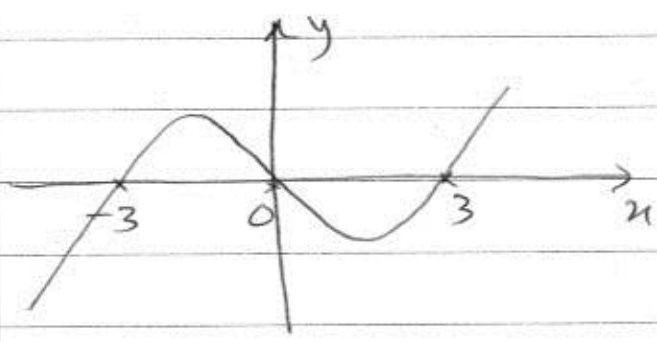


y-intercept: sub $x=0$

$$y = (-2)^3 = -8$$



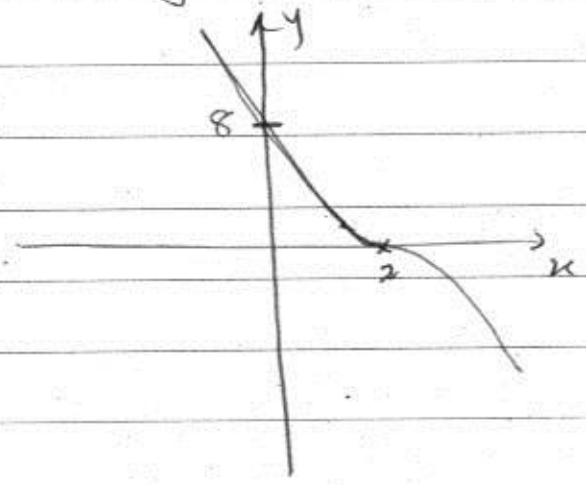
(i) $y = x^3 - 9x$



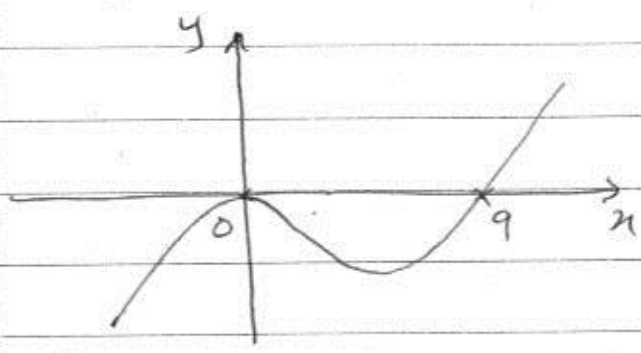
(b) $y = (2-x)^3$

Similar to Ex B / Q1 (a)

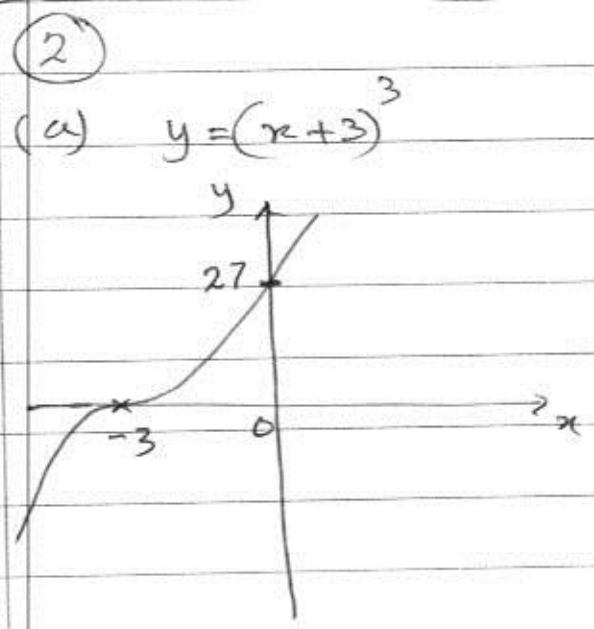
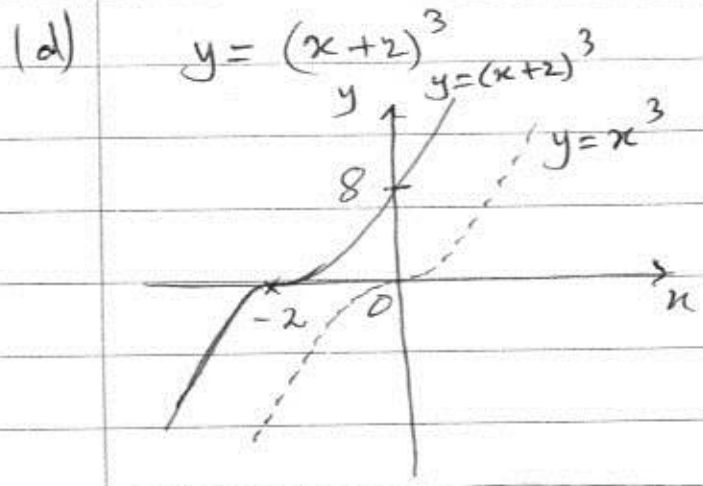
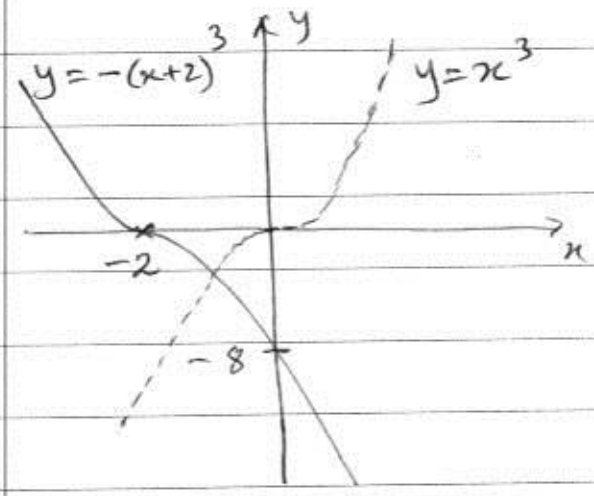
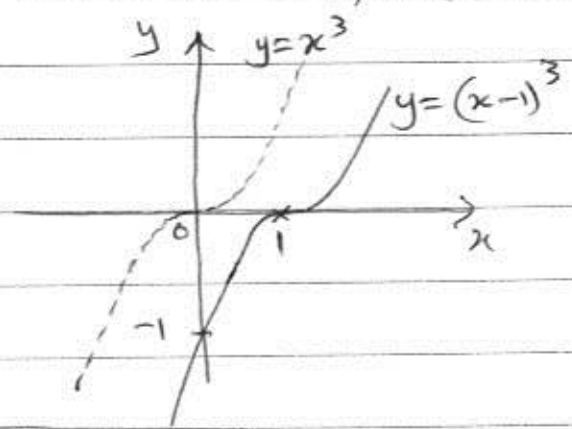
but negative.



(j) $y = x^3 - 9x^2$



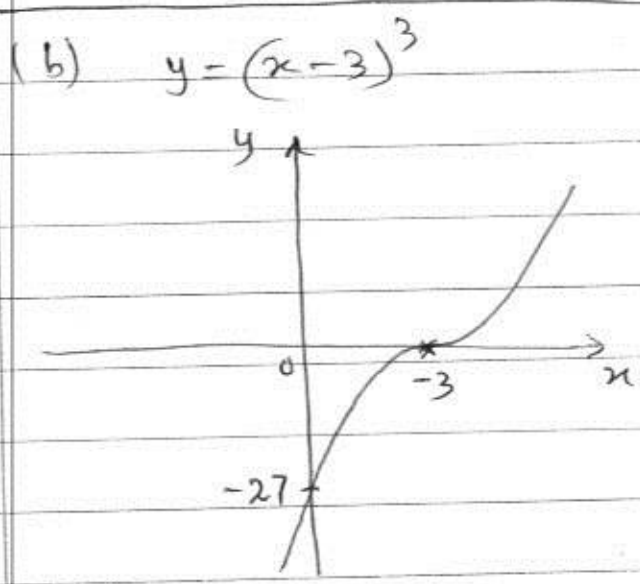
(c) $y = (x-1)^3$
Similar to Ex B / Q (a)



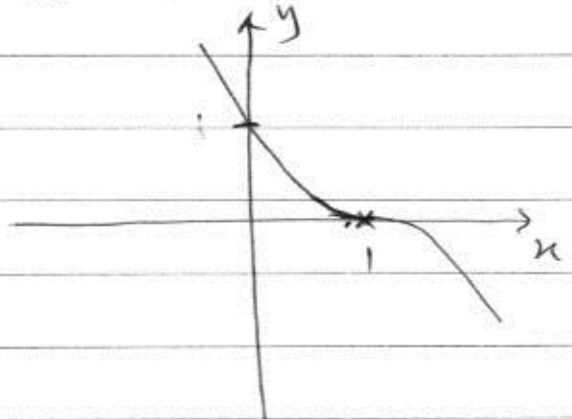
(e) $y = -(x+2)^3$
shape: $a = -1$

x-intercept: ~~at~~
 $-(x+2)^3 = 0$
 $(x+2)^3 = 0$
 $x = -2$

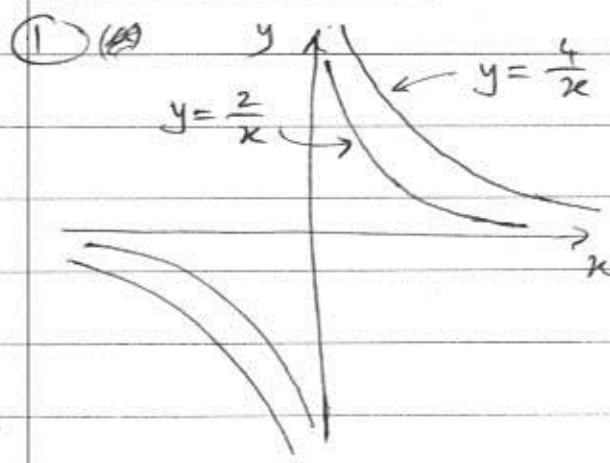
y-intercept: sub $x = 0$
 $y = -(2)^3$
 $= -8$



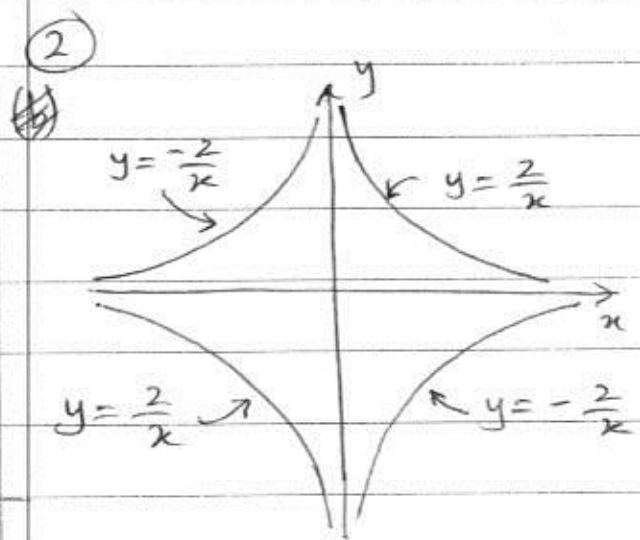
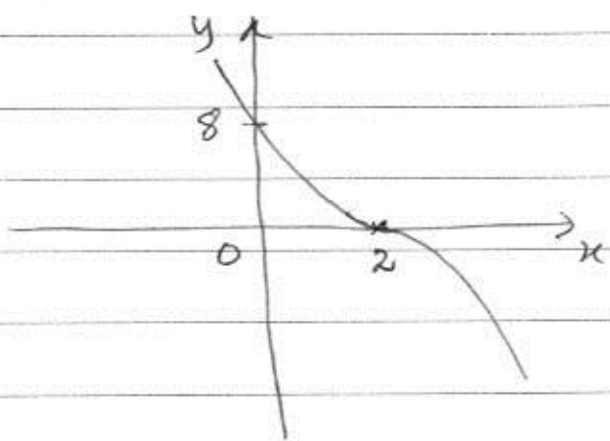
(c) $y = (1-x)^3$



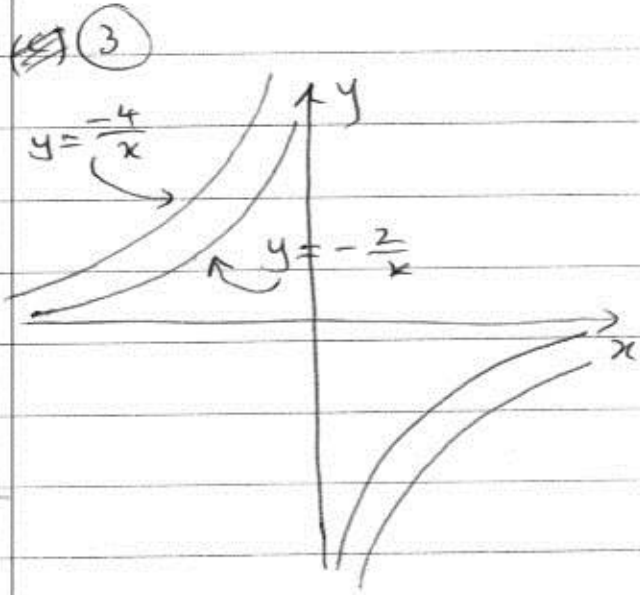
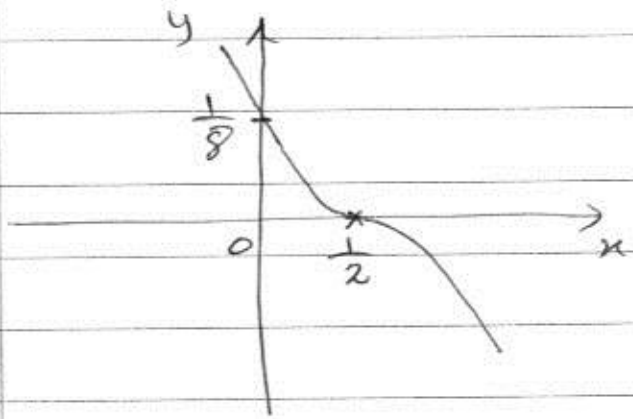
Exercise C



(d) $y = -(x-2)^3$



(e) $y = -(x - \frac{1}{2})^3$



④ Similar to Ex C / Q1

⑤ Similar to Ex C / Q3