Coordinate Geometry - Circles

Exercise A

1 Write down the equation of these circles:

  a Centre (3, 2), radius 4

  b Centre (−4, 5), radius 6

  c Centre (5, −6), radius $2\sqrt{3}$

  d Centre (2a, 7a), radius 5a

  e Centre (−2√2, −3√2), radius 1

2 Write down the coordinates of the centre and the radius of these circles:

  a $(x + 5)^2 + (y − 4)^2 = 9^2$

  b $(x − 7)^2 + (y − 1)^2 = 16$

  c $(x + 4)^2 + y^2 = 25$

  d $(x + 4a)^2 + (y + a)^2 = 144a^2$

  e $(x − 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27$

3 In each case, show that the circle passes through the given point:

  a $(x − 2)^2 + (y − 5)^2 = 13$, (4, 8)

  b $(x + 7)^2 + (y − 2)^2 = 65$, (0, −2)

  c $x^2 + y^2 = 25^2$, (7, −24)

  d $(x − 2a)^2 + (y + 5a)^2 = 20a^2$, (6a, −3a)

  e $(x − 3\sqrt{5})^2 + (y − \sqrt{5})^2 = (2\sqrt{10})^2$, (√5, −√5)

4 The point (4, −2) lies on the circle centre (8, 1). Find the equation of the circle.

5 The line PQ is the diameter of the circle, where P and Q are (5, 6) and (−2, 2) respectively. Find the equation of the circle.

6 The point (1, −3) lies on the circle $(x − 3)^2 + (y + 4)^2 = r^2$. Find the value of r.

7 The line $y = 2x + 13$ touches the circle $x^2 + (y − 3)^2 = 20$ at (−4, 5). Show that the radius at (−4, 5) is perpendicular to the line.

8 The line $x + 3y − 11 = 0$ touches the circle $(x + 1)^2 + (y + 6)^2 = 90$ at (2, 3).

  a Find the radius of the circle.

  b Show that the radius at (2, 3) is perpendicular to the line.

9 The point $P(1, −2)$ lies on the circle centre (4, 6).

  a Find the equation of the circle.

  b Find the equation of the tangent to the circle at P.

10 The tangent to the circle $(x + 4)^2 + (y − 1)^2 = 242$ at (7, −10) meets the y-axis at S and the x-axis at T.

  a Find the coordinates of S and T.

  b Hence, find the area of $\triangle OST$, where O is the origin.

Exercise B

1 Find where the circle $(x - 1)^2 + (y - 3)^2 = 45$ meets the x-axis.

2 Find where the circle $(x - 2)^2 + (y + 3)^2 = 29$ meets the y-axis.

3 The circle $(x - 3)^2 + (y + 3)^2 = 34$ meets the x-axis at $(a, 0)$ and the y-axis at $(0, b)$. Find the possible values of a and b.

4 The line $y = x + 4$ meets the circle $(x - 3)^2 + (y - 5)^2 = 34$ at A and B. Find the coordinates of A and B.

5 Find where the line $x + y + 5 = 0$ meets the circle $(x + 3)^2 + (y + 5)^2 = 65$.

6 Show that the line $y = x - 10$ does not meet the circle $(x - 2)^2 + y^2 = 25$.

7 Show that the line $x + y = 11$ is a tangent to the circle $x^2 + (y - 3)^2 = 32$.

> **Hint for question 7:**
> Show that the line meets the circle at one point only.

8 Show that the line $3x - 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$.

9 The line $y = 2x - 2$ meets the circle $(x - 2)^2 + (y - 2)^2 = 20$ at A and B.

   a Find the coordinates of A and B.

   b Show that AB is a diameter of the circle.

10 The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at $(3, 10)$, where a and p are constants.

   a Work out the value of a.   **b** Work out the two possible values of p.