Binomial Expansions (Year 12)

Exercise A

1 Write down the expansion of the following:

a $(2x+y)^4$ **b** $(p-q)^5$ **c** $(1+2x)^4$ **d** $(3+x)^4$

e $(1-\frac{1}{2}x)^4$ **f** $(4-x)^4$ **g** $(2x+3y)^5$ **h** $(x+2)^6$

2 Find the term in x^3 of the following expansions:

a $(3+x)^5$ **b** $(2x+y)^5$ **c** $(1-x)^6$ **d** $(3+2x)^5$

e $(1+x)^{10}$ **f** $(3-2x)^6$ **g** $(1+x)^{20}$ **h** $(4-3x)^7$

3 Use the binomial theorem to find the first four terms in the expansion of:

a $(1+x)^{10}$ **b** $(1-2x)^5$ **c** $(1+3x)^6$ **d** $(2-x)^8$

e $(2-\frac{1}{2}x)^{10}$ **f** $(3-x)^7$ **g** $(x+2y)^8$ **h** $(2x-3y)^9$

4 The coefficient of x^2 in the expansion of $(2+ax)^6$ is 60.
Find possible values of the constant a.

5 The coefficient of x^3 in the expansion of $(3+bx)^5$ is -720.
Find the value of the constant b.

6 The coefficient of x^3 in the expansion of $(2+x)(3-ax)^4$ is 30.
Find the values of the constant a.

7 Write down the first four terms in the expansion of $\left(1-\frac{x}{10}\right)^6$.

By substituting an appropriate value for x, find an approximate value to $(0.99)^6$. Use your calculator to find the degree of accuracy of your approximation.

8 Write down the first four terms in the expansion of $\left(2+\frac{x}{5}\right)^{10}$.

By substituting an appropriate value for x, find an approximate value to $(2.1)^{10}$. Use your calculator to find the degree of accuracy of your approximation.

Exercise B

1 Use the binomial expansion to find the first four terms of

a $(1+x)^8$ **b** $(1-2x)^6$ **c** $\left(1+\frac{x}{2}\right)^{10}$

d $(1-3x)^5$ **e** $(2+x)^7$ **f** $(3-2x)^3$

g $(2-3x)^6$ **h** $(4+x)^4$ **i** $(2+5x)^7$

2 If x is so small that terms of x^3 and higher can be ignored, show that:

$$(2+x)(1-3x)^5 \approx 2 - 29x + 165x^2$$

3 If x is so small that terms of x^3 and higher can be ignored, and

$$(2-x)(3+x)^4 \approx a + bx + cx^2$$

find the values of the constants a, b and c.

4 When $(1-2x)^p$ is expanded, the coefficient of x^2 is 40. Given that $p > 0$, use this information to find:

a The value of the constant p.

b The coefficient of x.

c The coefficient of x^3.

5 Write down the first four terms in the expansion of $(1+2x)^8$. By substituting an appropriate value of x (which should be stated), find an approximate value of 1.02^8. State the degree of accuracy of your answer.

Exercise C

1 When $(1-\frac{3}{2}x)^p$ is expanded in ascending powers of x, the coefficient of x is -24.

a Find the value of p.

b Find the coefficient of x^2 in the expansion.

c Find the coefficient of x^3 in the expansion. **E**

2 Given that:

$$(2-x)^{13} \equiv A + Bx + Cx^2 + \dots$$

Find the values of the integers A, B and C. **E**

3 **a** Expand $(1-2x)^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient in the expansion.

b Use your expansion to find an approximation to $(0.98)^{10}$, stating clearly the substitution which you have used for x. **E**

4 **a** Use the binomial series to expand $(2-3x)^{10}$ in ascending powers of x up to and including the term in x^3, giving each coefficient as an integer.

b Use your series expansion, with a suitable value for x, to obtain an estimate for 1.97^{10}, giving your answer to 2 decimal places. **E**

5 **a** Expand $(3+2x)^4$ in ascending powers of x, giving each coefficient as an integer.

b Hence, or otherwise, write down the expansion of $(3-2x)^4$ in ascending powers of x.

c Hence by choosing a suitable value for x show that $(3+2\sqrt{2})^4 + (3-2\sqrt{2})^4$ is an integer and state its value. **E**

6 The coefficient of x^2 in the binomial expansion of $\left(1 + \frac{x}{2}\right)^n$, where n is a positive integer, is 7.

a Find the value of n.

b Using the value of n found in part **a**, find the coefficient of x^4. *E*

7 **a** Use the binomial theorem to expand $(3 + 10x)^4$ giving each coefficient as an integer.

b Use your expansion, with an appropriate value for x, to find the exact value of $(1003)^4$. State the value of x which you have used. *E*

8 **a** Expand $(1 + 2x)^{12}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value for x, which must be stated, into your answer to part **a**, calculate an approximate value of $(1.02)^{12}$.

c Use your calculator, writing down all the digits in your display, to find a more exact value of $(1.02)^{12}$.

d Calculate, to 3 significant figures, the percentage error of the approximation found in part **b**. *E*

9 Expand $\left(x - \frac{1}{x}\right)^5$, simplifying the coefficients. *E*

10 In the binomial expansion of $(2k + x)^n$, where k is a constant and n is a positive integer, the coefficient of x^2 is equal to the coefficient of x^3.

a Prove that $n = 6k + 2$.

b Given also that $k = \frac{2}{3}$, expand $(2k + x)^n$ in ascending powers of x up to and including the term in x^3, giving each coefficient as an exact fraction in its simplest form. *E*

11 **a** Expand $(2 + x)^6$ as a binomial series in ascending powers of x, giving each coefficient as an integer.

b By making suitable substitutions for x in your answer to part **a**, show that $(2 + \sqrt{3})^6 - (2 - \sqrt{3})^6$ can be simplified to the form $k\sqrt{3}$, stating the value of the integer k. *E*

12 The coefficient of x^2 in the binomial expansion of $(2 + kx)^8$, where k is a positive constant, is 2800.

a Use algebra to calculate the value of k.

b Use your value of k to find the coefficient of x^3 in the expansion. *E*

13 **a** Given that

$$(2 + x)^5 + (2 - x)^5 \equiv A + Bx^2 + Cx^4,$$

find the value of the constants A, B and C.

b Using the substitution $y = x^2$ and your answers to part **a**, solve

$$(2 + x)^5 + (2 - x)^5 = 349.$$ *E*

14 In the binomial expansion of $(2 + px)^5$, where p is a constant, the coefficient of x^3 is 135. Calculate:

a The value of p,

b The value of the coefficient of x^4 in the expansion. *E*