Coordinate Geometry - Straight Line Graphs

Exercise A

1 Work out the gradient of the line joining these pairs of points:

a $(4, 2), (6, 3)$ **b** $(-1, 3), (5, 4)$

c $(-4, 5), (1, 2)$ **d** $(2, -3), (6, 5)$

e $(-3, 4), (7, -6)$ **f** $(-12, 3), (-2, 8)$

2 The line joining $(3, -5)$ to $(6, a)$ has gradient 4. Work out the value of a.

3 The line joining $(5, b)$ to $(8, 3)$ has gradient -3. Work out the value of b.

4 The line joining $(c, 4)$ to $(7, 6)$ has gradient $\frac{3}{4}$. Work out the value of c.

Exercise B

1. Find the gradient and y-intercept of each of the following lines.

a $y = -x + 4$ **b** $y = 2x - 5$ **c** $y = \frac{1}{2}x - \frac{2}{3}$

d $y = -3x$ **e** $y = \frac{6}{7}x + \frac{7}{5}$ **f** $y = 2 - 7x$

g $3x - 4y + 8 = 0$ **h** $4x - 5y - 10 = 0$ **i** $-2x + y - 9 = 0$

j $7x + 4y + 12 = 0$ **k** $7x - 2y + 3 = 0$ **l** $-5x + 4y + 2 = 0$

2.

Write these lines in the form $ax + by + c = 0$.

a $y = 4x + 3$ **b** $y = -3x - 2$ **c** $y = -6x + 7$

d $y = \frac{4}{5}x - 6$ **e** $y = \frac{5}{3}x + 2$ **f** $y = \frac{7}{3}x$

g $y = 2x - \frac{4}{7}$ **h** $y = -3x + \frac{2}{9}$ **i** $y = -6x - \frac{2}{3}$

j $y = -\frac{1}{3}x + \frac{1}{2}$ **k** $y = \frac{2}{3}x + \frac{5}{6}$ **l** $y = \frac{3}{5}x + \frac{1}{2}$

3.

A line is parallel to the line $3x + 6y + 11 = 0$ and its intercept on the y-axis is $(0, 7)$. Write down the equation of the line.

4.

The line $3x + 2y - 5 = 0$ meets the x-axis at the point R. Work out the coordinates of R.

5.

The line $5x - 4y + 20 = 0$ meets the y-axis at the point A and the x-axis at the point B. Work out the coordinates of the points A and B.

Exercise C

1 Find the equation of the line with gradient m that passes through the point (x_1, y_1) when:

a $m = 2$ and $(x_1, y_1) = (2, 5)$

b $m = 3$ and $(x_1, y_1) = (-2, 1)$

c $m = -1$ and $(x_1, y_1) = (3, -6)$

d $m = -4$ and $(x_1, y_1) = (-2, -3)$

e $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 10)$

f $m = -\frac{2}{3}$ and $(x_1, y_1) = (-6, -1)$

g $m = 2$ and $(x_1, y_1) = (a, 2a)$

h $m = -\frac{1}{2}$ and $(x_1, y_1) = (-2b, 3b)$

2 The line $y = 4x - 8$ meets the x-axis at the point A. Find the equation of the line with gradient 3 that passes through the point A.

3 The line $y = -2x + 8$ meets the y-axis at the point B. Find the equation of the line with gradient 2 that passes through the point B.

4 The line $y = \frac{1}{2}x + 6$ meets the x-axis at the point C. Find the equation of the line with gradient $\frac{2}{3}$ that passes through the point C. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

5 The line $y = \frac{1}{4}x + 2$ meets the y-axis at the point B. The point C has coordinates $(-5, 3)$. Find the gradient of the line joining the points B and C.

6 The lines $y = x$ and $y = 2x - 5$ intersect at the point A. Find the equation of the line with gradient $\frac{2}{5}$ that passes through the point A. (Hint: Solve $y = x$ and $y = 2x - 5$ simultaneously.)

7 The lines $y = 4x - 10$ and $y = x - 1$ intersect at the point T. Find the equation of the line with gradient $-\frac{2}{3}$ that passes through the point T. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

8 The line p has gradient $\frac{2}{3}$ and passes through the point $(6, -12)$. The line q has gradient -1 and passes through the point $(5, 5)$. The line p meets the y-axis at A and the line q meets the x-axis at B. Work out the gradient of the line joining the points A and B.

Exercise D

1 Find the equation of the line that passes through these pairs of points:

a $(2, 4)$ and $(3, 8)$

b $(0, 2)$ and $(3, 5)$

c $(-2, 0)$ and $(2, 8)$

d $(5, -3)$ and $(7, 5)$

2 The line that passes through the points $(2, -5)$ and $(-7, 4)$ meets the x-axis at the point P. Work out the coordinates of the point P.

3 The line that passes through the points $(-3, -5)$ and $(4, 9)$ meets the y-axis at the point G. Work out the coordinates of the point G.

4 The line that passes through the points $(3, 2\frac{1}{2})$ and $(-1\frac{1}{2}, 4)$ meets the y-axis at the point J. Work out the coordinates of the point J.

5 The line $y = 2x - 10$ meets the x-axis at the point A. The line $y = -2x + 4$ meets the y-axis at the point B. Find the equation of the line joining the points A and B. (Hint: First work out the coordinates of the points A and B.)

6 The line $y = 4x + 5$ meets the y-axis at the point C. The line $y = -3x - 15$ meets the x-axis at the point D. Find the equation of the line joining the points C and D. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

7 The lines $y = x - 5$ and $y = 3x - 13$ intersect at the point S. The point T has coordinates $(-4, 2)$. Find the equation of the line that passes through the points S and T.

8 The lines $y = -2x + 1$ and $y = x + 7$ intersect at the point L. The point M has coordinates $(-3, 1)$. Find the equation of the line that passes through the points L and M.

9 The vertices of the triangle ABC have coordinates $A(3, 5)$, $B(-2, 0)$ and $C(4, -1)$. Find the equations of the sides of the triangle.

10 The line V passes through the points $(-5, 3)$ and $(7, -3)$ and the line W passes through the points $(2, -4)$ and $(4, 2)$. The lines V and W intersect at the point A. Work out the coordinates of the point A.

Exercise E

1.

Find an equation of the line:

a parallel to the line $y = -2x - 5$, passing through $(-\frac{1}{2}, \frac{3}{2})$

b parallel to the line $x - 2y - 1 = 0$, passing through $(0, 0)$

c perpendicular to the line $y = x - 4$, passing through $(-1, -2)$

d perpendicular to the line $2x + y - 9 = 0$, passing through $(4, -6)$.

2.

Find an equation of the line:

a parallel to the line $y = 3x + 6$, passing through $(-2, 5)$

b perpendicular to the line $y = 3x + 6$, passing through $(-2, 5)$

c parallel to the line $4x - 6y + 7 = 0$, passing through $(3, 4)$

d perpendicular to the line $4x - 6y + 7 = 0$, passing through $(3, 4)$.

3.

The line r passes through the points $(1, 4)$ and $(6, 8)$ and the line s passes through the points $(5, -3)$ and $(20, 9)$. Show that the lines r and s are parallel.

4.

The vertices of a quadrilateral $ABCD$ has coordinates $A(-1, 5)$, $B(7, 1)$, $C(5, -3)$, $D(-3, 1)$. Show that the quadrilateral is a rectangle.

Exercise F

1 The points A and B have coordinates $(-4, 6)$ and $(2, 8)$ respectively. A line p is drawn through B perpendicular to AB to meet the y-axis at the point C.

a Find an equation of the line p.

b Determine the coordinates of C. E

2 The line l has equation $2x - y - 1 = 0$.
The line m passes through the point $A(0, 4)$ and is perpendicular to the line l.

a Find an equation of m and show that the lines l and m intersect at the point $P(2, 3)$.

The line n passes through the point $B(3, 0)$ and is parallel to the line m.

b Find an equation of n and hence find the coordinates of the point Q where the lines l and n intersect. E

3 The line L_1 has gradient $\frac{1}{7}$ and passes through the point $A(2, 2)$. The line L_2 has gradient -1 and passes through the point $B(4, 8)$. The lines L_1 and L_2 intersect at the point C.

a Find an equation for L_1 and an equation for L_2.

b Determine the coordinates of C. E

4 The straight line passing through the point $P(2, 1)$ and the point $Q(k, 11)$ has gradient $-\frac{5}{12}$.

a Find the equation of the line in terms of x and y only.

b Determine the value of k. E

5 **a** Find an equation of the line l which passes through the points $A(1, 0)$ and $B(5, 6)$.

The line m with equation $2x + 3y = 15$ meets l at the point C.

b Determine the coordinates of the point C. E

6 The line L passes through the points $A(1, 3)$ and $B(-19, -19)$.
Find an equation of L in the form $ax + by + c = 0$, where a, b and c are integers. E

7 The straight line l_1 passes through the points A and B with coordinates $(2, 2)$ and $(6, 0)$ respectively.

a Find an equation of l_1.

The straight line l_2 passes through the point C with coordinates $(-9, 0)$ and has gradient $\frac{1}{4}$.

b Find an equation of l_2. E

8 The straight line l_1 passes through the points A and B with coordinates $(0, -2)$ and $(6, 7)$ respectively.

a Find the equation of l_1 in the form $y = mx + c$.

The straight line l_2 with equation $x + y = 8$ cuts the y-axis at the point C. The lines l_1 and l_2 intersect at the point D.

b Calculate the coordinates of the point D.

c Calculate the area of $\triangle ACD$. E

9 The points A and B have coordinates $(2, 16)$ and $(12, -4)$ respectively. A straight line l_1 passes through A and B.

a Find an equation for l_1 in the form $ax + by = c$.

The line l_2 passes through the point C with coordinates $(-1, 1)$ and has gradient $\frac{1}{3}$.

b Find an equation for l_2. E

10 The points $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\triangle ABC$. Angle ABC is a right angle.

a Find the gradient of AB.

b Calculate the value of k.

c Find an equation of the straight line passing through B and C. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. E

11 The straight line l passes through $A(1, 3\sqrt{3})$ and $B(2 + \sqrt{3}, 3 + 4\sqrt{3})$.

a Calculate the gradient of l giving your answer as a surd in its simplest form.

b Give the equation of l in the form $y = mx + c$, where constants m and c are surds given in their simplest form.

c Show that l meets the x-axis at the point $C(-2, 0)$. E

12 **a** Find an equation of the straight line passing through the points with coordinates $(-1, 5)$ and $(4, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

The line crosses the x-axis at the point A and the y-axis at the point B, and O is the origin.

b Find the area of $\triangle OAB$. E

13 The points A and B have coordinates $(k, 1)$ and $(8, 2k - 1)$ respectively, where k is a constant. Given that the gradient of AB is $\frac{1}{3}$:

a show that $k = 2$

b find an equation for the line through A and B. E

14 The straight line l_1 has equation $4y + x = 0$.
The straight line l_2 has equation $y = 2x - 3$.

a On the same axes, sketch the graphs of l_1 and l_2. Show clearly the coordinates of all points at which the graphs meet the coordinate axes.

The lines l_1 and l_2 intersect at the point A.

b Calculate, as exact fractions, the coordinates of A.

c Find an equation of the line through A which is perpendicular to l_1. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. E

15 The points A and B have coordinates $(4, 6)$ and $(12, 2)$ respectively.
The straight line l_1 passes through A and B.

a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The straight line l_2 passes through the origin and has gradient -4.

b Write down an equation for l_2.

The lines l_1 and l_2 intersect at the point C.

c Find the coordinates of C. E