Binomial Expansions – Year 13

1. Find the first four terms in the expansion of each of the following functions, stating the range of values of x for which the expansion is valid.

(i) $(1+x)^{-2}$ **(ii)** $\frac{1}{1+2x}$ **(iii)** $\sqrt{1-x^2}$

(iv) $\frac{1+2x}{1-2x}$ **(v)** $(3+x)^{-1}$ **(vi)** $(1-x)\sqrt{4+x}$

(vii) $\frac{x+2}{x-3}$ **(viii)** $\frac{1}{\sqrt{3x+4}}$ **(ix)** $\frac{1+2x}{(2x-1)^2}$

(x) $\frac{1+x^2}{1-x^2}$ **(xi)** $\sqrt[3]{1+2x^2}$ **(xii)** $\frac{1}{(1+2x)(1+x)}$

2.

(i) Write down the expansion of $(1+x)^3$.

(ii) Find the first four terms in the expansion of $(1-x)^{-4}$ in ascending powers of x. For what values of x is this expansion valid?

(iii) When the expansion is valid

$$\frac{(1+x)^3}{(1-x)^4} = 1 + 7x + ax^2 + bx^3 + \ldots .$$

Find the values of a and b. [MEI]

3.

3 (i) Write down the expansion of $(2-x)^4$.

(ii) Find the first four terms in the expansion of $(1+2x)^{-3}$ in ascending powers of x. For what range of values of x is this expansion valid?

(iii) When the expansion is valid

$$\frac{(2-x)^4}{(1+2x)^3} = 16 + ax + bx^2 + \ldots .$$

Find the values of a and b. [MEI]

4 Write down the expansions of the following expressions in ascending powers of x, as far as the term containing x^3. In each case state the values of x for which the expansion is valid.

(i) $(1-x)^{-1}$ **(ii)** $(1+2x)^{-2}$ **(iii)** $\frac{1}{(1-x)(1+2x)^2}$

[MEI]

5 (i) Show that $\frac{1}{\sqrt{4-x}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$.

(ii) Write down the first three terms in the binomial expansion of $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ in ascending powers of x, stating the range of values of x for which this expansion is valid.

(iii) Find the first three terms in the expansion of $\frac{2(1+x)}{\sqrt{4-x}}$ in ascending powers of x, for small values of x.

[MEI]

6 (i) Expand $(1+y)^{-1}$, where $-1 < y < 1$, as a series in powers of y, giving the first four terms.

(ii) Hence find the first four terms of the expansion of $\left(1+\frac{2}{x}\right)^{-1}$ where $-1 < \frac{2}{x} < 1$.

(iii) Show that $\left(1+\frac{2}{x}\right)^{-1} = \frac{x}{x+2} = \frac{x}{2}\left(1+\frac{x}{2}\right)^{-1}$.

(iv) Find the first four terms of the expansion of $\frac{x}{2}\left(1+\frac{x}{2}\right)^{-1}$ where $-1 < \frac{x}{2} < 1$.

(v) State the conditions on x under which your expansions for $\left(1+\frac{2}{x}\right)^{-1}$ and $\frac{x}{2}\left(1+\frac{x}{2}\right)^{-1}$ are valid and explain briefly why your expansions are different.

[MEI]

7 (i) Use integration by parts to show that

$$\int_1^k t\ln t\,\mathrm{d}t = \tfrac{1}{2}k^2\ln k - \tfrac{1}{4}k^2 + \tfrac{1}{4}$$

where k is positive.

(ii) Expand $(1-2x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3, giving your answer in simplified form.
State the range of values of x for which the expansion is valid.

(iii) Hence show that, provided x is small, $(1-2x)^{-\frac{1}{2}}\ln(1+x)$ is approximately equal to $t\ln t$, where $t = 1+x$.
Hence find an approximate value for $\int_0^{0.1} \frac{\ln(1+x)}{\sqrt{1-2x}}\,\mathrm{d}x$.

[MEI]