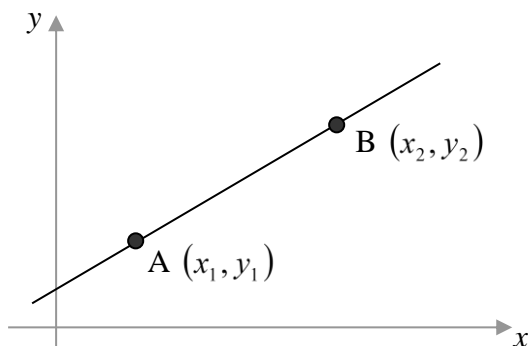# Coordinate Geometry – Straight Lines

1. Finding the gradient of a line, if the coordinates of two points on the line are given
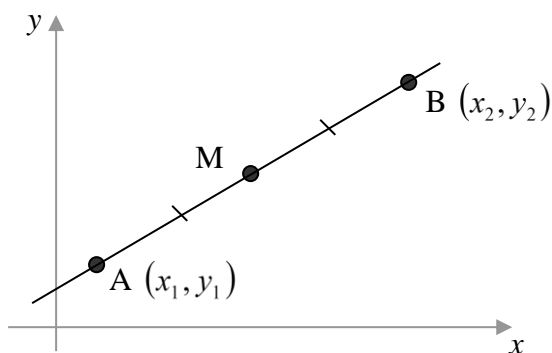

$$\text{Gradient, m} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1
A line passes through the points A (3, -8) and B (5, 2). Find the gradient of the line.

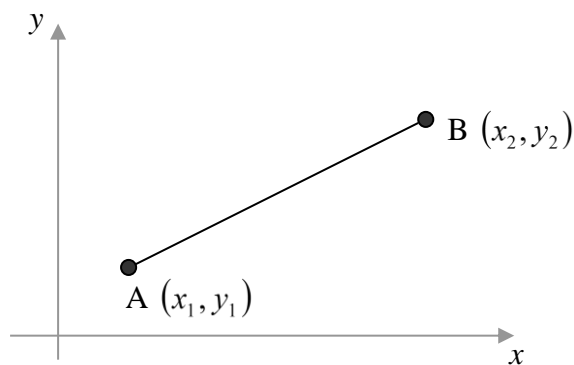**2. Finding the coordinates of the mid-point of two given points**

$$\text{Mid-point, M} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example 2
Find the coordinates of the mid-point of the line segment joining A (3, -8) and B (5, 2).

3. Finding the distance between two points, if the coordinates of the points are given.

$$\text{Distance AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 3
Find the length of the line segment joining A (3, -8) and B (5, 2).

---

4. The $ax + by + c = 0$ form for straight line equations

Equations of sloped straight lines are usually written in the form $y = mx + c$. However, sometimes these equations are written in the form $ax + by + c = 0$. You should be able to work out the gradient and y-intercept if the equation of a straight line is given in the form $ax + by + c = 0$. This is done by rearranging the equation into the form, $y = mx + c$.

Example 4
Find the gradient and y-intercept of the line with equation, $12x + 7y - 3 = 0$.

---

5. Finding the equation of a line, if the gradient of the line and the coordinates of one point on the line are given.
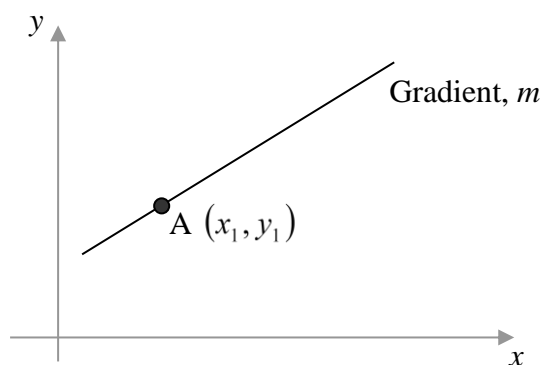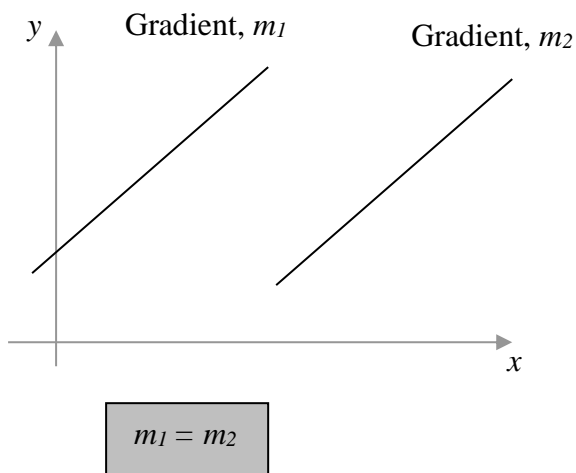

$$\text{The equation of the line is, } y - y_1 = m(x - x_1)$$

Example 5
The gradient of a line is 2. The line passes through the point A, which has coordinates (3, -1). Find the equation of the line.
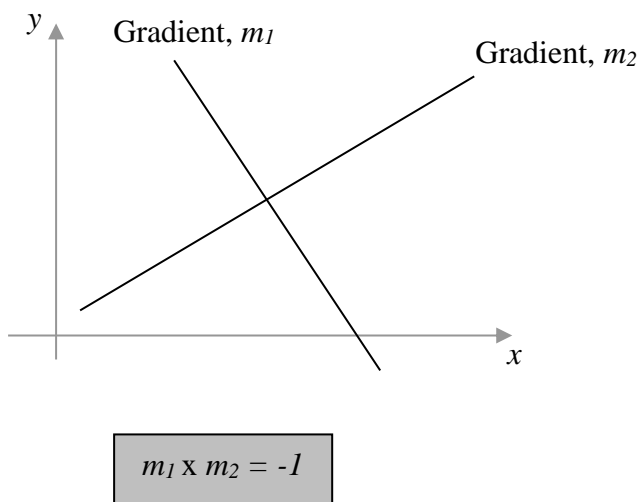
6. Gradients of Parallel lines

Parallel lines have the same gradient.

$$m_1 = m_2$$

7. Gradient of Perpendicular lines

Gradients of perpendicular lines multiply to give -1.

$$m_1 \text{ x } m_2 = -1$$

8. Finding the coordinates of the point of intersection of two straight lines

To find the coordinates of the point of intersection of two lines, you should solve the equations of the lines simultaneously.

Example 6

Find the coordinates of the point where the lines $y = 2x + 3$ and $y = 3x + 1$ intersect.

> **Note**
>
> Not only for straight lines, but also for any two graphs, you may solve the equations of the graphs simultaneously, in order to find the coordinates of the point(s) of intersection.

9. Finding the coordinates of the points, where a straight line crosses the x and y axes

- To find the x coordinate of the point where a line crosses the x-axis, substitute $y = 0$ in the equation of the line and work out x.

- To find the y coordinate of the point where a line crosses the y-axis, substitute $x = 0$ in the equation of the line and work out y. Alternatively you may find the value of the y-intercept (c).

> **Note**
>
> Not only for straight lines, but also for any two graphs, you may use the same methods to find the coordinates of the point, where the graphs cross the x and y axes. Recall that we learnt this in sketching graphs.