Integration – Year 12

Exercise A

1 Find the following integrals:

a $\int (2x+3)x^2\,dx$ **b** $\int \frac{(2x^2+3)}{x^2}\,dx$ **c** $\int (2x+3)^2\,dx$

d $\int (2x+3)(x-1)\,dx$ **e** $\int (2x+3)\sqrt{x}\,dx$

2 Find $\int f(x)dx$ when $f(x)$ is given by the following:

a $(x+2)^2$ **b** $\left(x+\frac{1}{x}\right)^2$ **c** $(\sqrt{x}+2)^2$

d $\sqrt{x}(x+2)$ **e** $\left(\frac{x+2}{\sqrt{x}}\right)$ **f** $\left(\frac{1}{\sqrt{x}}+2\sqrt{x}\right)$

3 Find the following integrals:

a $\int\left(3\sqrt{x}+\frac{1}{x^2}\right)dx$ **b** $\int\left(\frac{2}{\sqrt{x}}+3x^2\right)dx$

c $\int\left(x^{\frac{2}{3}}+\frac{4}{x^3}\right)dx$ **d** $\int\left(\frac{2+x}{x^3}+3\right)dx$

e $\int (x^2+3)(x-1)dx$ **f** $\int\left(\frac{2}{\sqrt{x}}+3x\sqrt{x}\right)dx$

g $\int (x-3)^2\,dx$ **h** $\int \frac{(2x+1)^2}{\sqrt{x}}\,dx$

i $\int\left(3+\frac{\sqrt{x}+6x^3}{x}\right)dx$ **j** $\int \sqrt{x}(\sqrt{x}+3)^2\,dx$

Exercise B

1 Evaluate the following definite integrals:

a $\int_1^2 \left(\frac{2}{x^3}+3x\right)dx$ **b** $\int_0^2 (2x^3-4x+5)\,dx$

c $\int_4^9 \left(\sqrt{x}-\frac{6}{x^2}\right)dx$ **d** $\int_1^2 \left(6x-\frac{12}{x^4}+3\right)dx$

e $\int_1^8 (x^{-\frac{1}{3}}+2x-1)\,dx$

2 Evaluate the following definite integrals:

a $\int_1^3 \left(\frac{x^3+2x^2}{x}\right)dx$ **b** $\int_1^4 (\sqrt{x}-3)^2\,dx$ **c** $\int_3^6 \left(x-\frac{3}{x}\right)^2 dx$

d $\int_0^1 x^2\left(\sqrt{x}+\frac{1}{x}\right)dx$ **e** $\int_1^4 \frac{2+\sqrt{x}}{x^2}\,dx$

Exercise C

1 Find the equation of the curve with the given derivative of y with respect to x that passes through the given point:

a $\frac{dy}{dx} = 3x^2 + 2x$; point (2, 10)

b $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$; point (1, 4)

c $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$; point (4, 11)

d $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$; point (4, 0)

e $\frac{dy}{dx} = (x + 2)^2$; point (1, 7)

f $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}}$; point (0, 1)

2 The curve C, with equation $y = f(x)$, passes through the point (1, 2) and $f'(x) = 2x^3 - \frac{1}{x^2}$. Find the equation of C in the form $y = f(x)$.

3 The gradient of a particular curve is given by $\frac{dy}{dx} = \frac{\sqrt{x} + 3}{x^2}$. Given that the curve passes through the point (9, 0), find an equation of the curve.

4 A set of curves, that each pass through the origin, have equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$ ..., where $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = x^2$.

a Find $f_2(x)$, $f_3(x)$.

b Suggest an expression for $f_n(x)$.

5 A set of curves, with equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$ ... all pass through the point (0, 1) and they are related by the property $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = 1$.
Find $f_2(x)$, $f_3(x)$, $f_4(x)$.

Exercise D

1 Find the area between the curve with equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ in each of the following cases:

a $f(x) = 3x^2 - 2x + 2$; $a = 0$, $b = 2$

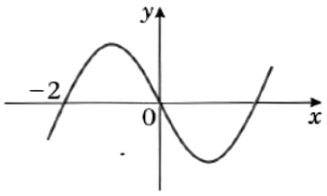
b $f(x) = x^3 + 4x$; $a = 1$, $b = 2$

c $f(x) = \sqrt{x} + 2x$; $a = 1$, $b = 4$

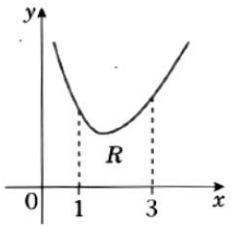
d $f(x) = 7 + 2x - x^2$; $a = -1$, $b = 2$

e $f(x) = \frac{8}{x^3} + \sqrt{x}$; $a = 1$, $b = 4$

2 The sketch shows part of the curve with equation $y = x(x^2 - 4)$.
Find the area of the shaded region.



3 The diagram shows a sketch of the curve with equation $y = 3x + \frac{6}{x^2} - 5, x > 0$.
The region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$.
Find the area of R.



4 Find the area of the finite region between the curve with equation $y = (3 - x)(1 + x)$ and the x-axis.

5 Find the area of the finite region between the curve with equation $y = x(x - 4)^2$ and the x-axis.

6 Find the area of the finite region between the curve with equation $y = x^2(2 - x)$ and the x-axis.

Exercise E

Sketch the following and find the area of the finite region or regions bounded by the curves and the x-axis:

1 $y = x(x + 2)$

2 $y = (x + 1)(x - 4)$

3 $y = (x + 3)x(x - 3)$

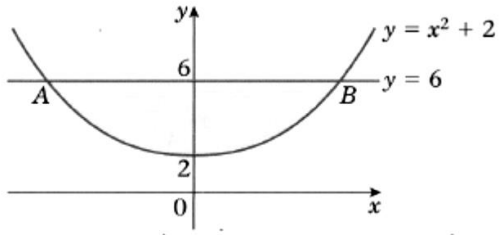
4 $y = x^2(x - 2)$

5 $y = x(x - 2)(x - 5)$

(Exercise F is on the next page)

Exercise F

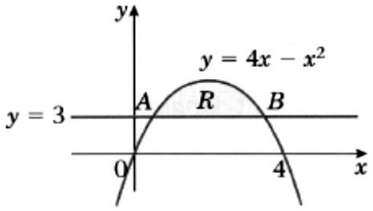
1 The diagram shows part of the curve with equation $y = x^2 + 2$ and the line with equation $y = 6$. The line cuts the curve at the points A and B.



a Find the coordinates of the points A and B.

b Find the area of the finite region bounded by AB and the curve.

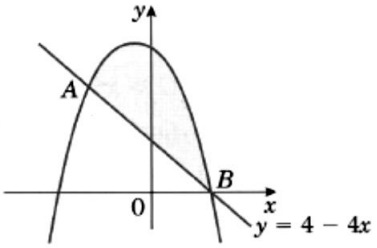
2 The diagram shows the finite region, R, bounded by the curve with equation $y = 4x - x^2$ and the line $y = 3$. The line cuts the curve at the points A and B.



a Find the coordinates of the points A and B.

b Find the area of R.

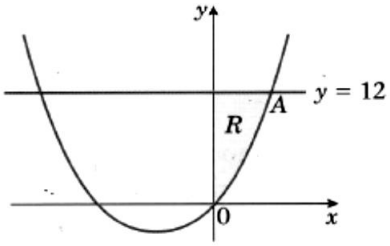
3 The diagram shows a sketch of part of the curve with equation $y = 9 - 3x - 5x^2 - x^3$ and the line with equation $y = 4 - 4x$. The line cuts the curve at the points $A\ (-1, 8)$ and $B\ (1, 0)$.



Find the area of the shaded region between AB and the curve.

4 Find the area of the finite region bounded by the curve with equation $y = (1 - x)(x + 3)$ and the line $y = x + 3$.

5 The diagram shows the finite region, R, bounded by the curve with equation $y = x(4 + x)$, the line with equation $y = 12$ and the y-axis.

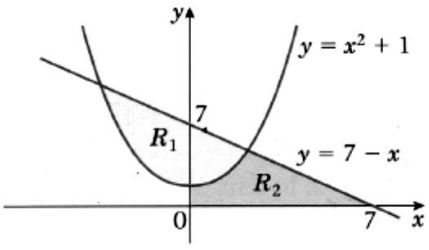


a Find the coordinate of the point A where the line meets the curve.

b Find the area of R.

(This exercise continues on the next page)

6 The diagram shows a sketch of part of the curve with equation $y = x^2 + 1$ and the line with equation $y = 7 - x$. The finite region R_1 is bounded by the line and the curve. The finite region R_2 is below the curve and the line and is bounded by the positive x- and y-axes as shown in the diagram.



a Find the area of R_1.

b Find the area of R_2.

7 The curve C has equation $y = x^{\frac{2}{3}} - \dfrac{2}{x^{\frac{1}{3}}} + 1$.

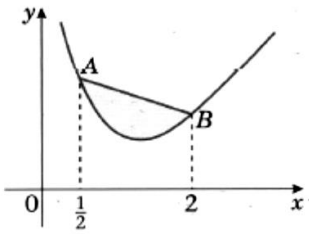
a Verify that C crosses the x-axis at the point (1, 0).

b Show that the point A (8, 4) also lies on C.

c The point B is (4, 0). Find the equation of the line through AB.
The finite region R is bounded by C, AB and the positive x-axis.

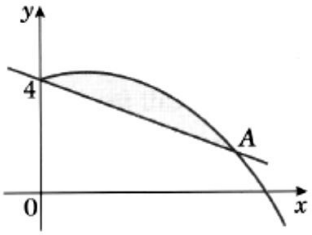
d Find the area of R.

8 The diagram shows part of a sketch of the curve with equation $y = \dfrac{2}{x^2} + x$. The points A and B have x-coordinates $\frac{1}{2}$ and 2 respectively.



Find the area of the finite region between AB and the curve.

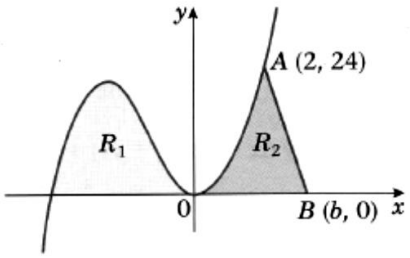
9 The diagram shows part of the curve with equation $y = 3\sqrt{x} - \sqrt{x^3} + 4$ and the line with equation $y = 4 - \frac{1}{2}x$.



a Verify that the line and the curve cross at the point A (4, 2).

b Find the area of the finite region bounded by the curve and the line.

10 The sketch shows part of the curve with equation $y = x^2(x + 4)$. The finite region R_1 is bounded by the curve and the negative x-axis. The finite region R_2 is bounded by the curve, the positive x-axis and AB, where A (2, 24) and B (b, 0).



The area of R_1 = the area of R_2.

a Find the area of R_1.

b Find the value of b.