Revision Paper 6
(Due on 14 March 2020)

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1. A geometric series has first term a and common ratio $r = \dfrac{3}{4}$

The sum of the first 4 terms of this series is 175

(a) Show that $a = 64$

(2)

(b) Find the sum to infinity of the series.

(2)

(c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

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2. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leqslant x \leqslant 4$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 \left(8 - 2^{x-1}\right) \mathrm{d}x$

(3)

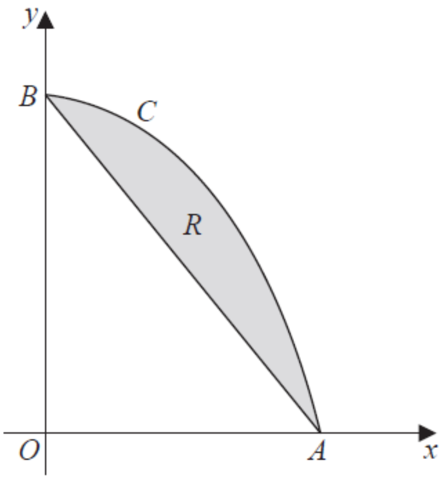

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}$, $\quad 0 \leqslant x \leqslant 4$

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

3.

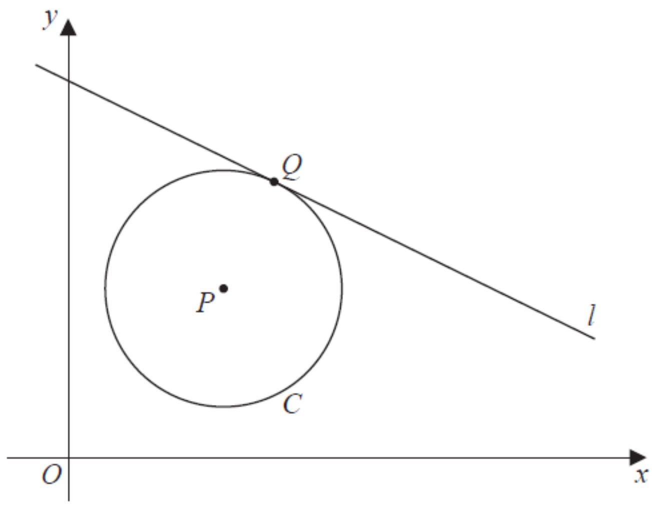

Diagram not drawn to scale

Figure 2

The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

4.

$$\mathrm{f}(x) = 6x^3 + 13x^2 - 4$$

(a) Use the remainder theorem to find the remainder when $\mathrm{f}(x)$ is divided by $(2x + 3)$.

(2)

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $\mathrm{f}(x)$.

(2)

(c) Factorise $\mathrm{f}(x)$ completely.

(4)

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5. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2-9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

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6. (i) Solve, for $-\pi < \theta \leqslant \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leqslant x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0$$

giving your answers to one decimal place.
(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

Question 7 is on the next page

7.

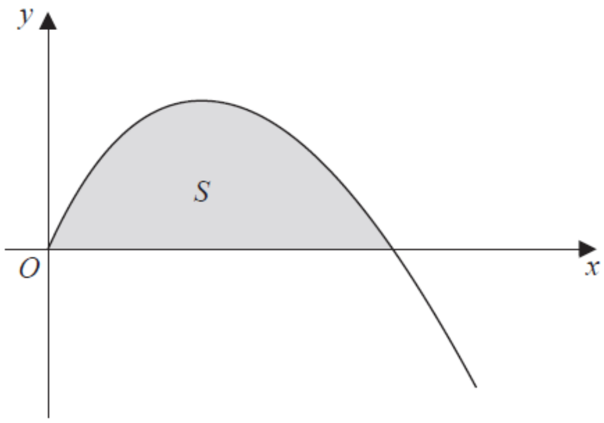

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x$$

(3)

(b) Hence find the area of S.

(3)

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8. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Question 9 is on the next page

9.

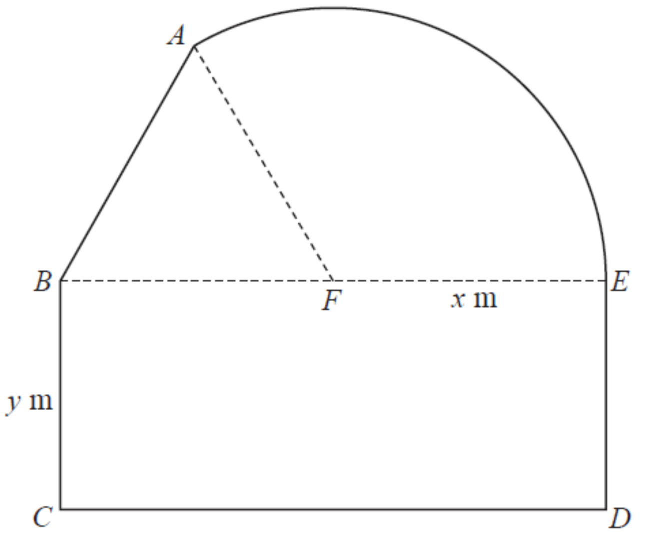

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.
(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\,m^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.
(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.
(2)