Motion with Variable Acceleration 1

1.

Given that $x = t^3 + 4t + 6$, find expressions for v and a in terms of t. Find the displacement, velocity and acceleration when $t = 2$.

2.

Given that $x = 120 - 15t - 6t^2 + t^3$, find the time when the velocity is zero. Find the displacement at this instant.

3.

A train leaves a station and travels in a straight line. After t seconds the train has travelled a distance x metres, where $x = \left(320t^3 - 2t^4\right) \times 10^{-5}$. This formula is valid until the train comes to rest at the next station.

(a) Find when the train comes to rest, and hence find the distance between the two stations.

(b) Find the acceleration of the train 40 seconds after the journey begins.

(c) Find the deceleration of the train just before it stops.

(d) Find when the acceleration is zero, and hence find the maximum velocity of the train.

4.

A particle moves in a straight line. Its velocity t s after leaving a fixed point on the line is v m s^{-1}, where $v = t + 0.1t^2$. Find

(i) an expression for the acceleration of the particle at time t, [2]

(ii) the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is 2.8 m s^{-2}. [7]

5.

A motorcyclist starts from rest at a point O and travels in a straight line. His velocity after t seconds is v m s^{-1}, for $0 \leqslant t \leqslant T$, where $v = 7.2t - 0.45t^2$. The motorcyclist's acceleration is zero when $t = T$.

(i) Find the value of T. [4]

(ii) Show that $v = 28.8$ when $t = T$. [1]

For $t \geqslant T$ the motorcyclist travels in the same direction as before, but with constant speed 28.8 m s^{-1}.

(iii) Find the displacement of the motorcyclist from O when $t = 31$. [6]

6.

A cyclist travels along a straight road. Her velocity $v\,\text{m s}^{-1}$, at time t seconds after starting from a point O, is given by

$$v = 2 \quad \text{for } 0 \leqslant t \leqslant 10,$$
$$v = 0.03t^2 - 0.3t + 2 \quad \text{for } t \geqslant 10.$$

(i) Find the displacement of the cyclist from O when $t = 10$. [1]

(ii) Show that, for $t \geqslant 10$, the displacement of the cyclist from O is given by the expression $0.01t^3 - 0.15t^2 + 2t + 5$. [4]

(iii) Find the time when the acceleration of the cyclist is $0.6\,\text{m s}^{-2}$. Hence find the displacement of the cyclist from O when her acceleration is $0.6\,\text{m s}^{-2}$. [5]

7.

A particle starts from rest at a point A at time $t = 0$, where t is in seconds. The particle moves in a straight line. For $0 \leqslant t \leqslant 4$ the acceleration is $1.8t\ \text{m s}^{-2}$, and for $4 \leqslant t \leqslant 7$ the particle has constant acceleration $7.2\,\text{m s}^{-2}$.

(i) Find an expression for the velocity of the particle in terms of t, valid for $0 \leqslant t \leqslant 4$. [3]

(ii) Show that the displacement of the particle from A is 19.2 m when $t = 4$. [4]

(iii) Find the displacement of the particle from A when $t = 7$. [5]

8.

A particle starts from rest at the point A and travels in a straight line. The displacement s m of the particle from A at time t s after leaving A is given by

$$s = 0.001t^4 - 0.04t^3 + 0.6t^2, \quad \text{for } 0 \leqslant t \leqslant 10.$$

(i) Show that the velocity of the particle is $4\,\text{m s}^{-1}$ when $t = 10$. [3]

The acceleration of the particle for $t \geqslant 10$ is $(0.8 - 0.08t)\,\text{m s}^{-2}$.

(ii) Show that the velocity of the particle is zero when $t = 20$. [5]

(iii) Find the displacement from A of the particle when $t = 20$. [6]

9.

A particle moves along the x-axis with velocity, $v\,\text{m s}^{-1}$, at time t given by

$$v = 24t - 6t^2.$$

The positive direction is in the sense of x increasing.

(i) Find an expression for the acceleration of the particle at time t. [2]

(ii) Find the times, t_1 and t_2, at which the particle has zero speed. [2]

(iii) Find the distance travelled between the times t_1 and t_2. [4]

(iv) Find the distance travelled between the times t_1 and $t = 7$ seconds [2]

10.

A toy car is travelling in a straight horizontal line.

One model of the motion for $0 \leqslant t \leqslant 8$, where t is the time in seconds, is shown in the velocity–time graph Fig. 6.

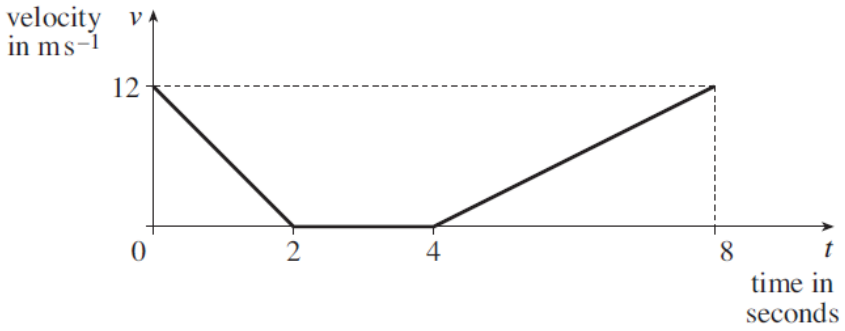

Fig. 6

(i) Calculate the distance travelled by the car from $t = 0$ to $t = 8$. [2]

(ii) How much less time would the car have taken to travel this distance if it had maintained its initial speed throughout? [1]

(iii) What is the acceleration of the car when $t = 1$? [2]

From $t = 8$ to $t = 14$, the car travels 58.5 m with a new constant acceleration, a m s^{-2}.

(iv) Find a. [2]

A second model for the velocity, v m s^{-1}, of the toy car is

$$v = 12 - 10t + \tfrac{9}{4}t^2 - \tfrac{1}{8}t^3, \text{ for } 0 \leqslant t \leqslant 8.$$

This model agrees with the values for v given in Fig. 6 for $t = 0, 2, 4$ and 6. [Note that you are not required to verify this.] Use this second model to answer the following questions.

(v) Calculate the acceleration of the car when $t = 1$. [3]

(vi) Initially the car is at A. Find an expression in terms of t for the displacement of the car from A after the first t seconds of its motion.

Hence find the displacement of the car from A when $t = 8$. [5]

(vii) Explain with a reason what this model predicts for the motion of the car between $t = 2$ and $t = 4$. [3]